

CSE167, Introduction to Computer Graphics  
Midterm Exam, Thursday November 8, 2007

Matthias Zwicker

Please include all steps of your derivations in your answers to show your understanding of the problem. Try not to write more than the recommended amount of text. If your answer is a mix of correct and substantially wrong arguments we will consider deducting points for incorrect statements. There are twelve questions for a total score of 100 points.

**Your name:**

1. Given two vectors  $(2, 1, 4)$  and  $(0, 2, 3)$ . Compute a unit vector that is perpendicular to these two vectors. **6 points**
2. Write down a  $4 \times 4$  homogeneous transformation matrix that performs a translation by a vector  $(a, b, c, 0)$ . Also write down the inverse of this matrix. **6 points**

3. Write down a  $4 \times 4$  homogeneous transformation matrix that performs a rotation by 90 degrees around the axis  $(0, 0, 1, 0)$  going through the origin  $(0, 0, 0, 1)$ . To verify that your matrix does the right thing, make sure that any point  $(0, 0, z, 1)$  for any value of  $z$  is not moved by your matrix. **8 points**

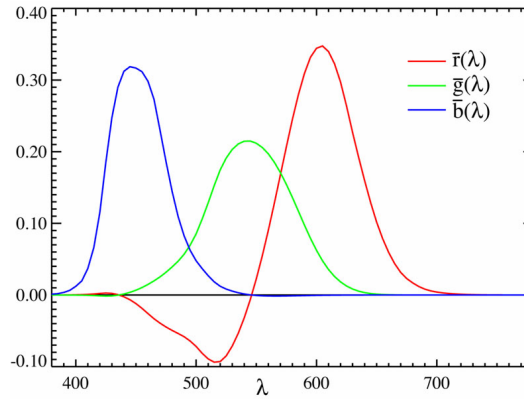
4. Write down a  $4 \times 4$  homogeneous transformation matrix that performs a rotation by 90 degrees around the axis  $(0, 0, 1, 0)$  going through a point  $(a, b, 0, 1)$ . To verify that your matrix does the right thing, make sure that any point  $(a, b, z, 1)$  for any value of  $z$  is not moved by your matrix. **8 points**

5. Assume you are working with three coordinate systems: object, world, and camera space. The basis vectors of object space have world coordinates  $(1, 0, 0)$ ,  $(0, 0, -1)$ , and  $(0, 1, 0)$ . The origin of object space has world coordinates  $(0, 0, 10)$ . The basis vectors of camera space have world coordinates  $(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$ ,  $(-1/\sqrt{2}, 0, 1/\sqrt{2})$ , and  $(1/\sqrt{6}, -2/\sqrt{6}, 1/\sqrt{6})$ . The origin of camera space has world coordinates  $(5, 9, -4)$ . Write down the  $4 \times 4$  matrices that transform object to world coordinates, camera to world coordinates, world to camera coordinates, and object to camera coordinates. For the object to camera coordinate transformation, you do not need to multiply out the matrix product. **12 points**

6. Given two points in the two-dimensional plane,  $\mathbf{p}_0 = (2, 1)$  and  $\mathbf{p}_1 = (4, 3)$ , that define a line. Write down the implicit line equation. Remember that this is a scalar-valued function  $f(\mathbf{p})$ ,  $\mathbf{p} \in \mathbf{R}^2$  that returns the signed distance of a point to the line. This means  $f(\mathbf{p})$  is zero for all points  $\mathbf{p}$  on the line, negative for all points on one side of the line, and positive for all points on the other side. **6 points**
7. Given a triangle in the two-dimensional plane with vertices  $\mathbf{a} = (1, 1)$ ,  $\mathbf{b} = (4, 2)$ , and  $\mathbf{c} = (3, 3)$ . Determine the barycentric coordinates of the point  $\mathbf{p} = (3, 2)$ . Write down the criteria to determine whether a point is inside a triangle using barycentric coordinates. Is  $\mathbf{p}$  inside the triangle  $\mathbf{a}, \mathbf{b}, \mathbf{c}$ ? **10 points**

8. Given a triangle that has been projected to the image plane. The  $w$  coordinates of its vertices **a**, **b**, and **c** are  $a_w = 4$ ,  $b_w = 4$ , and  $c_w = 8$ . The texture coordinates  $u, v$  of its vertices are  $a_{u,v} = (0, 0)$ ,  $b_{u,v} = (1, 0)$ , and  $c_{u,v} = (1, 1)$ . Assume that the rasterizer is currently visiting a pixel with barycentric coordinates  $\alpha = 1/2$ ,  $\beta = 1/8$ , and  $\gamma = 3/8$ . Compute the linear and the perspective correct interpolation of the texture coordinates at  $\alpha, \beta, \gamma$ . **10 points**
9. What is the purpose of  $z$ -buffering? Describe in two sentences. Write down pseudo-code for the  $z$ -buffer test. **8 points**

10. The figure below shows the *CIE RGB* matching curves. Describe how these curves were determined. Your answer should include an explanation of the meaning of the three values  $r(\lambda)$ ,  $g(\lambda)$ ,  $b(\lambda)$  for any given wavelength  $\lambda$ . It should also explain how to interpret negative values. (5-6 sentences) **10 points**



11. The CIE RGB color space is *not* perceptually uniform. Explain what this means and why this may be a disadvantage. (3-4 sentences) **8 points**

12. Assume that there is a point light at position  $\mathbf{p}$  that is illuminating a point  $\mathbf{s}$  on a surface. Explain why the strength of the light is proportional to  $1/\|\mathbf{p} - \mathbf{s}\|^2$ , i.e., one over the squared distance between the light and the point on a surface. (3-4 sentences) **8 points**