

CSE 167:
Introduction to Computer Graphics
Lecture #10: Scene Management

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Announcements

- ▶ Homework assignment #4 due Friday, Oct 29
- ▶ Problem #2 not due until Friday, Nov 5
- ▶ Two options for problem #4 (extra credit)

Lecture Overview

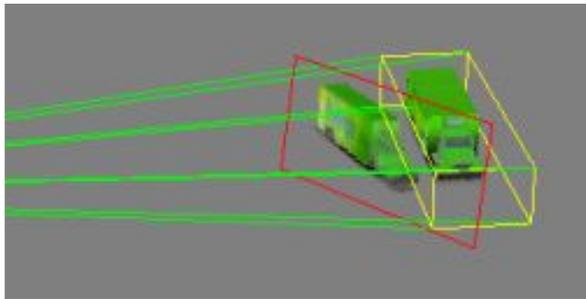
- ▶ Scene Graphs & Hierarchies
 - ▶ Performance Optimization
- ▶ Curves
 - ▶ Introduction
 - ▶ Polynomial curves

Performance Optimization

- ▶ **Level-of-detail techniques**
 - ▶ Use lower quality for distant (small) objects
- ▶ **Culling**
 - ▶ Quickly discard invisible parts of the scene
- ▶ **Scene graph compilation**
 - ▶ Efficient use of low-level API
 - ▶ Avoid state changes in rendering pipeline
 - ▶ Render objects with similar properties (geometry, shaders, materials) in batches

Level-of-Detail Techniques

- ▶ Don't draw objects smaller than a threshold
 - ▶ Popping artifacts
- ▶ Replace objects by impostors
 - ▶ Textured planes representing the objects



Dynamic impostor generation



Original vs. impostor

Level-of-Detail Techniques

- ▶ Adapt triangle count to projected size



With bump mapping

Without bump mapping

Culling

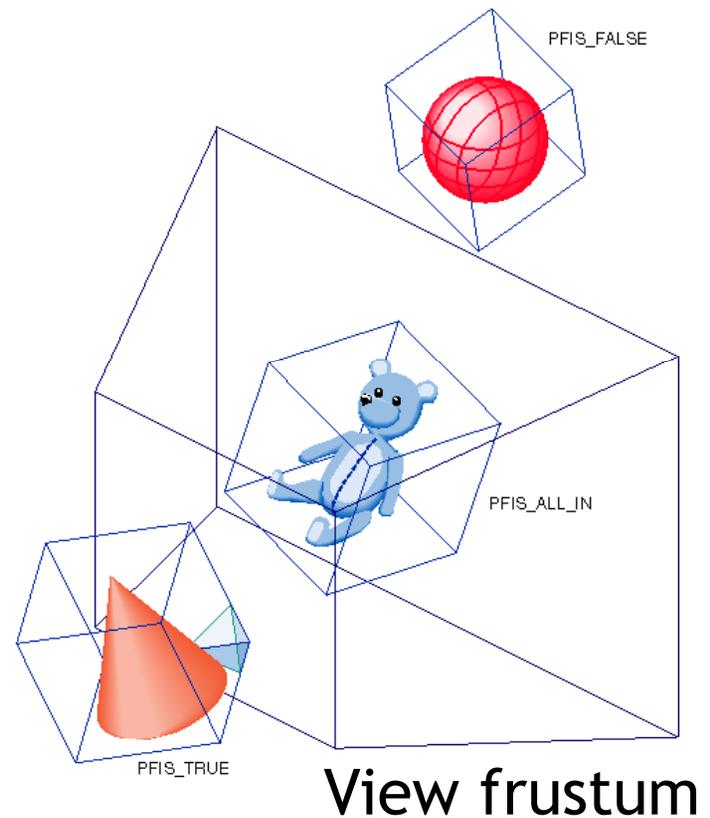
- ▶ **Occlusion culling**
 - ▶ Discard objects that are within view frustum, but hidden behind other objects
- ▶ **View frustum culling**
 - ▶ Discard objects outside view frustum
- ▶ **Essential for interactive performance with large scenes**

Occlusion Culling

- ▶ **Cell-based occlusion culling**
 - ▶ Divide scene into cells
 - ▶ Determine *potentially visible set* (PVS) for each cell
 - ▶ Discard all cells not in PVS
- ▶ **Two main variants**
 - ▶ Precomputation using binary space partitioning (BSP) trees
 - ▶ Portal algorithms
- ▶ **Specialized algorithms for different types of geometry**
 - ▶ Indoor scenes
 - ▶ Terrain

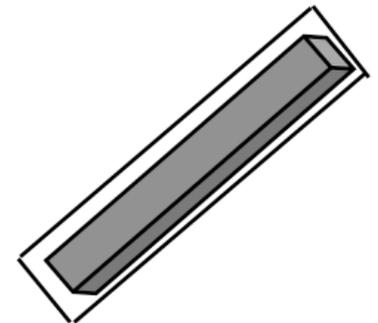
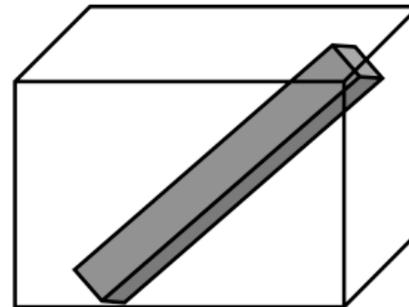
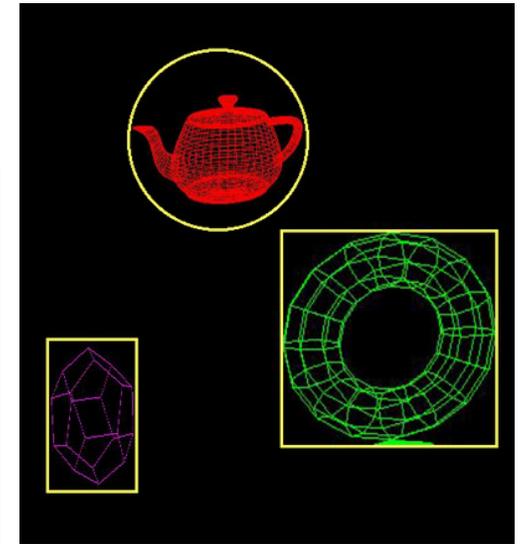
View Frustum Culling

- ▶ Frustum defined by 6 planes
- ▶ Each plane divides space into “outside”, “inside”
- ▶ Check each object against each plane
 - ▶ Outside, inside, intersecting
- ▶ If “outside” all planes
 - ▶ Outside the frustum
- ▶ If “inside” all planes
 - ▶ Inside the frustum
- ▶ Else partly inside and partly out
- ▶ Efficiency



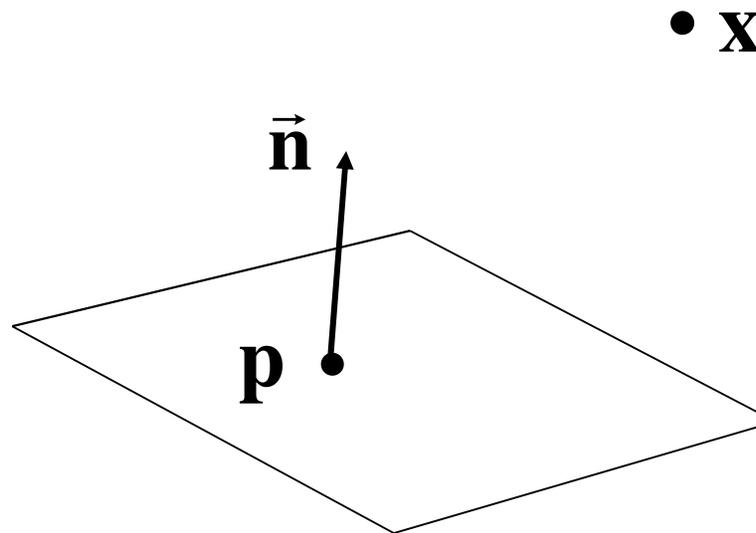
Bounding Volumes

- ▶ Simple shape that completely encloses an object
- ▶ Generally a box or sphere
- ▶ We use spheres
 - ▶ Easiest to work with
 - ▶ Though hard to get tight fits
- ▶ Intersect bounding volume with view frustum, instead of full geometry



Distance to Plane

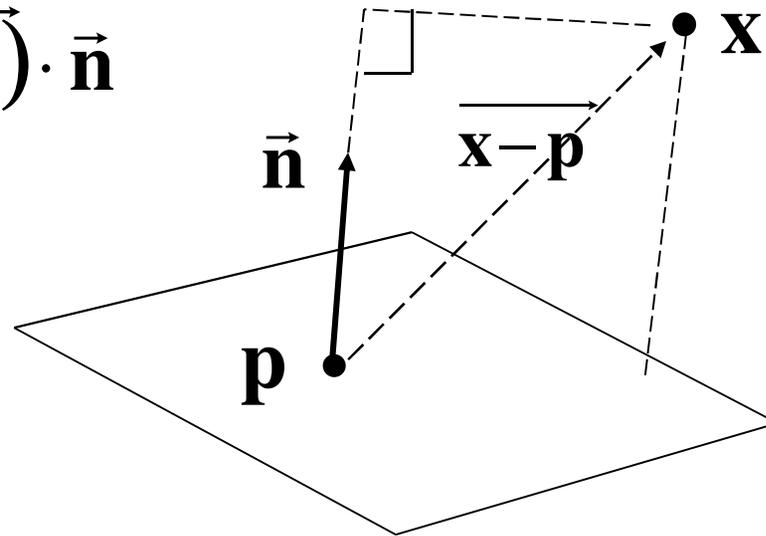
- ▶ A plane is described by a point \mathbf{p} on the plane and a unit normal \mathbf{n}
- ▶ Find the (perpendicular) distance from point \mathbf{x} to the plane



Distance to Plane

- ▶ The distance is the length of the projection of $\mathbf{x}-\mathbf{p}$ onto \mathbf{n}

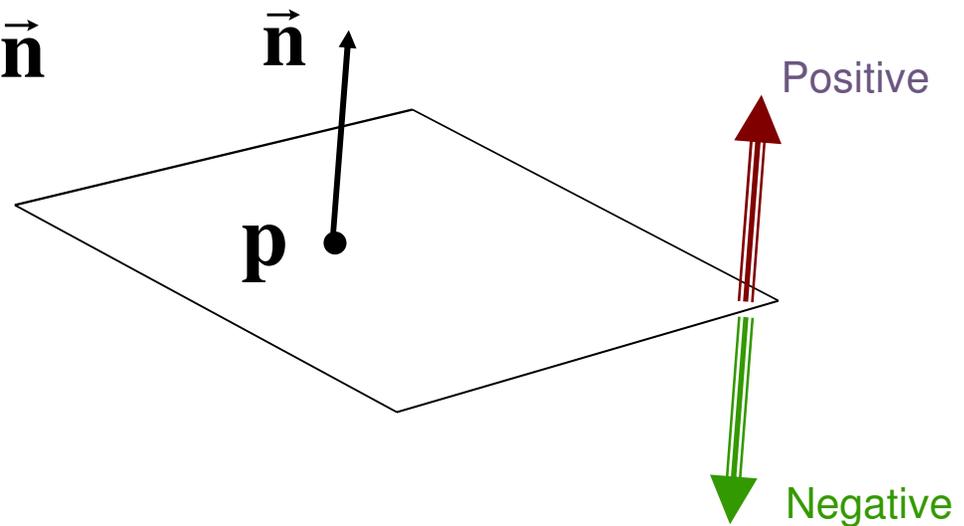
$$dist = \overrightarrow{(\mathbf{x} - \mathbf{p})} \cdot \vec{\mathbf{n}}$$



Distance to Plane

- ▶ The distance has a sign
 - ▶ positive on the side of the plane the normal points to
 - ▶ negative on the opposite side
 - ▶ zero exactly on the plane
- ▶ Divides 3D space into two infinite half-spaces

$$dist(\mathbf{x}) = \overrightarrow{(\mathbf{x} - \mathbf{p})} \cdot \vec{\mathbf{n}}$$



Distance to Plane

- ▶ Simplification

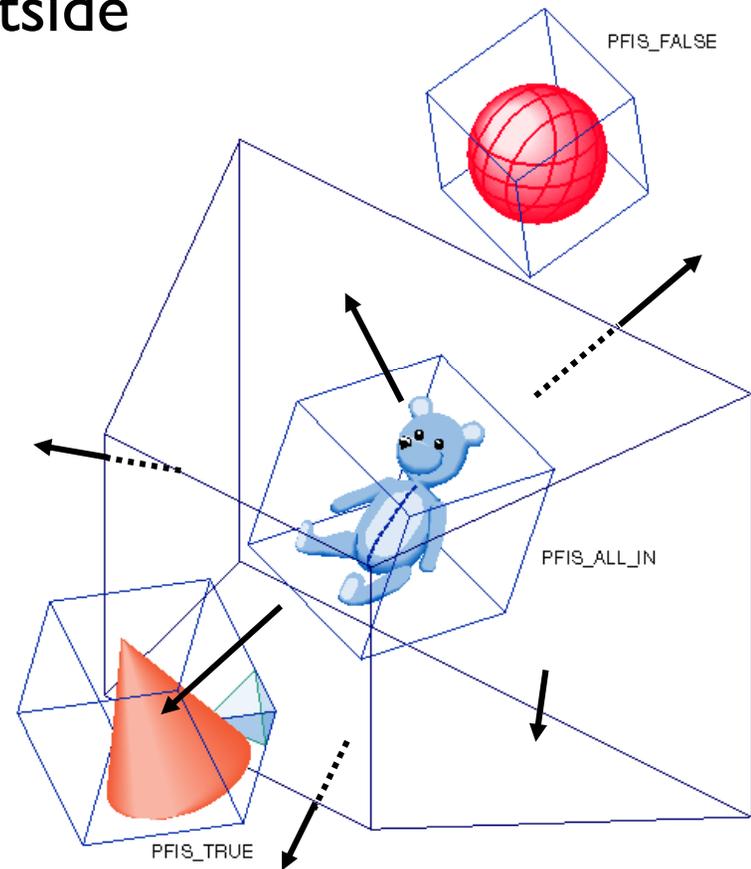
$$\begin{aligned}dist(\mathbf{x}) &= (\mathbf{x} - \mathbf{p}) \cdot \mathbf{n} \\ &= \mathbf{x} \cdot \mathbf{n} - \mathbf{p} \cdot \mathbf{n}\end{aligned}$$

$$dist(\mathbf{x}) = \mathbf{x} \cdot \mathbf{n} - d, \quad d = \mathbf{p} \cdot \mathbf{n}$$

- ▶ d is independent of \mathbf{x}
- ▶ d is distance from the origin to the plane
- ▶ We can represent a plane with just d and \mathbf{n}

Frustum With Signed Planes

- ▶ Normal of each plane points outside
 - ▶ “outside” means positive distance
 - ▶ “inside” means negative distance



Test Sphere and Plane

- ▶ For sphere with radius r and origin \mathbf{x} , test the distance to the origin, and see if it is beyond the radius

- ▶ Three cases:

- ▶ $dist(\mathbf{x}) > r$

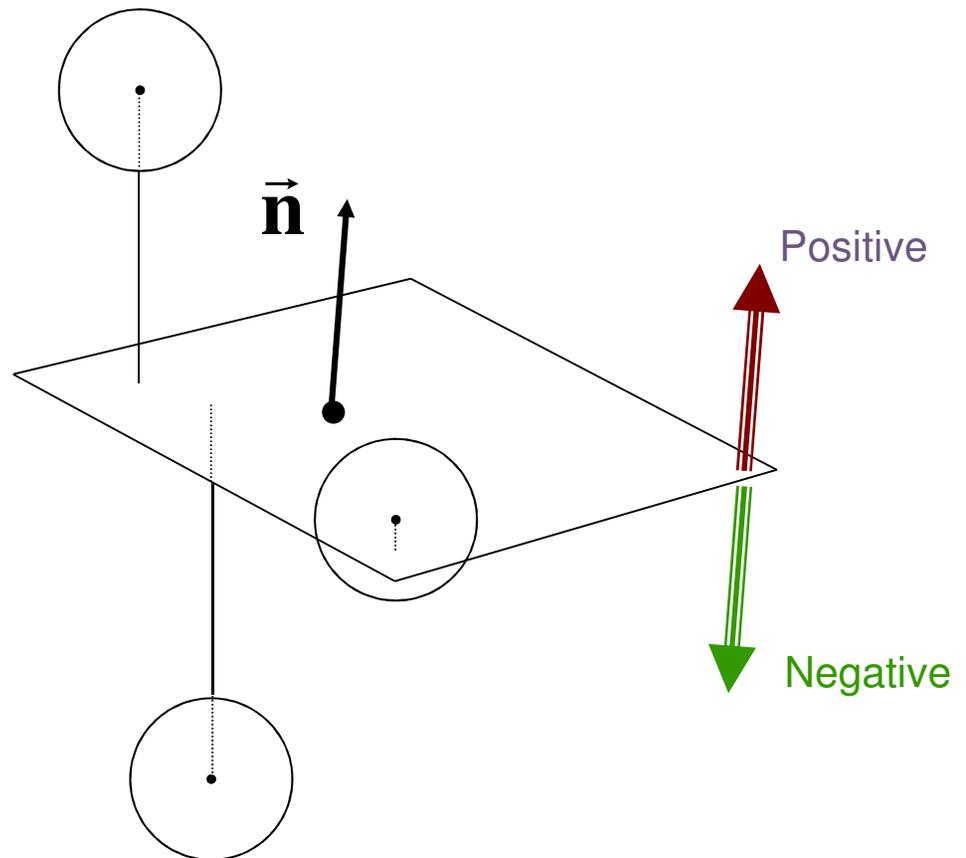
- ▶ completely above

- ▶ $dist(\mathbf{x}) < -r$

- ▶ completely below

- ▶ $-r < dist(\mathbf{x}) < r$

- ▶ intersects

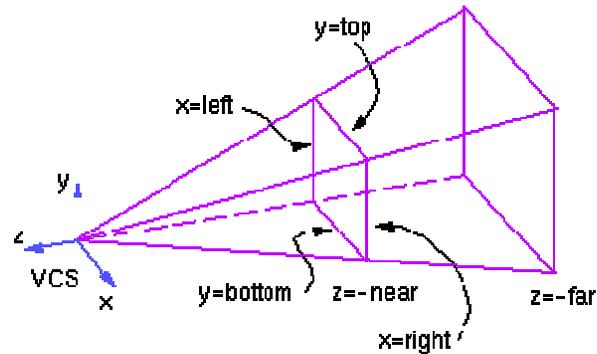


Culling Summary

- ▶ Precompute the normal \mathbf{n} and value d for each of the six planes.
- ▶ Given a sphere with center \mathbf{x} and radius r
- ▶ For each plane:
 - ▶ if $dist(\mathbf{x}) > r$: sphere is outside! (no need to continue loop)
 - ▶ add 1 to count if $dist(\mathbf{x}) < -r$
- ▶ If we made it through the loop, check the count:
 - ▶ if the count is 6, the sphere is completely inside
 - ▶ otherwise the sphere intersects the frustum
 - ▶ (*can use a flag instead of a count*)

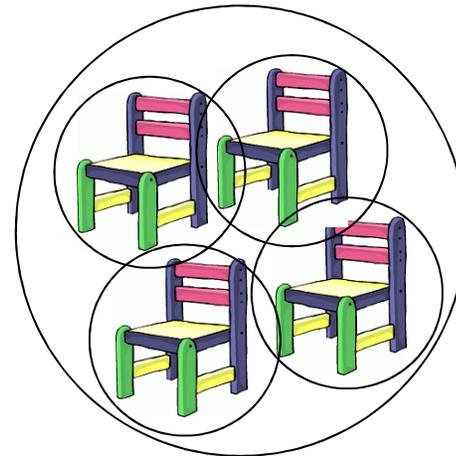
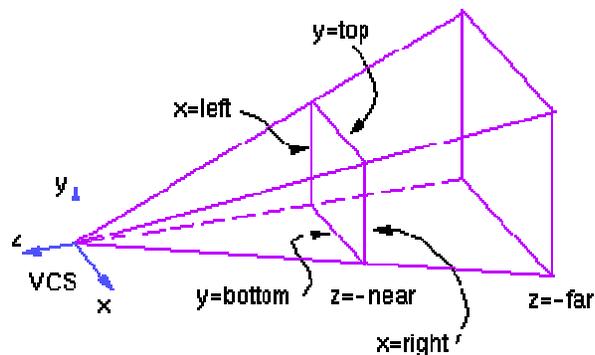
Culling Groups of Objects

- ▶ Want to be able to cull the whole group quickly
- ▶ But if the group is partly in and partly out, want to be able to cull individual objects



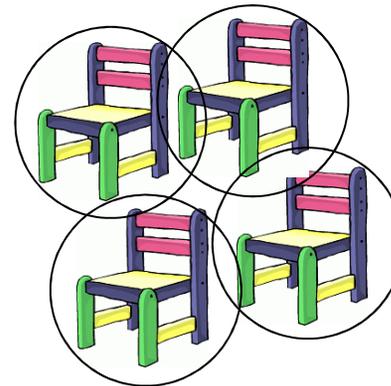
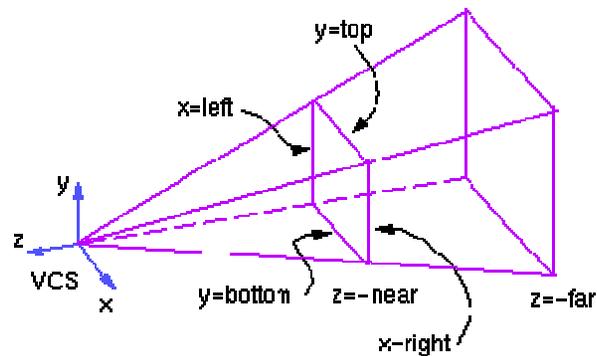
Hierarchical Bounding Volumes

- ▶ Given hierarchy of objects
- ▶ Bounding volume of each node encloses the bounding volumes of all its children
- ▶ Start by testing the outermost bounding volume
 - ▶ If it is entirely outside, don't draw the group at all
 - ▶ If it is entirely inside, draw the whole group



Hierarchical Culling

- ▶ If the bounding volume is partly inside and partly outside
 - ▶ Test each child's bounding volume individually
 - ▶ If the child is in, draw it; if it's out cull it; if it's partly in and partly out, recurse.
 - ▶ If recursion reaches a leaf node, draw it normally

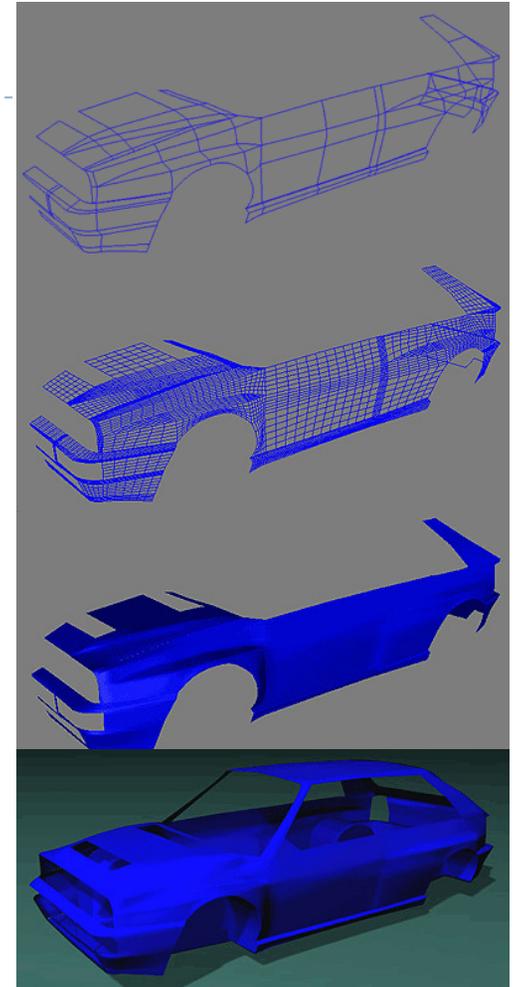


Lecture Overview

- ▶ Scene Graphs & Hierarchies
 - ▶ Performance Optimization
- ▶ Curves
 - ▶ Introduction
 - ▶ Polynomial curves

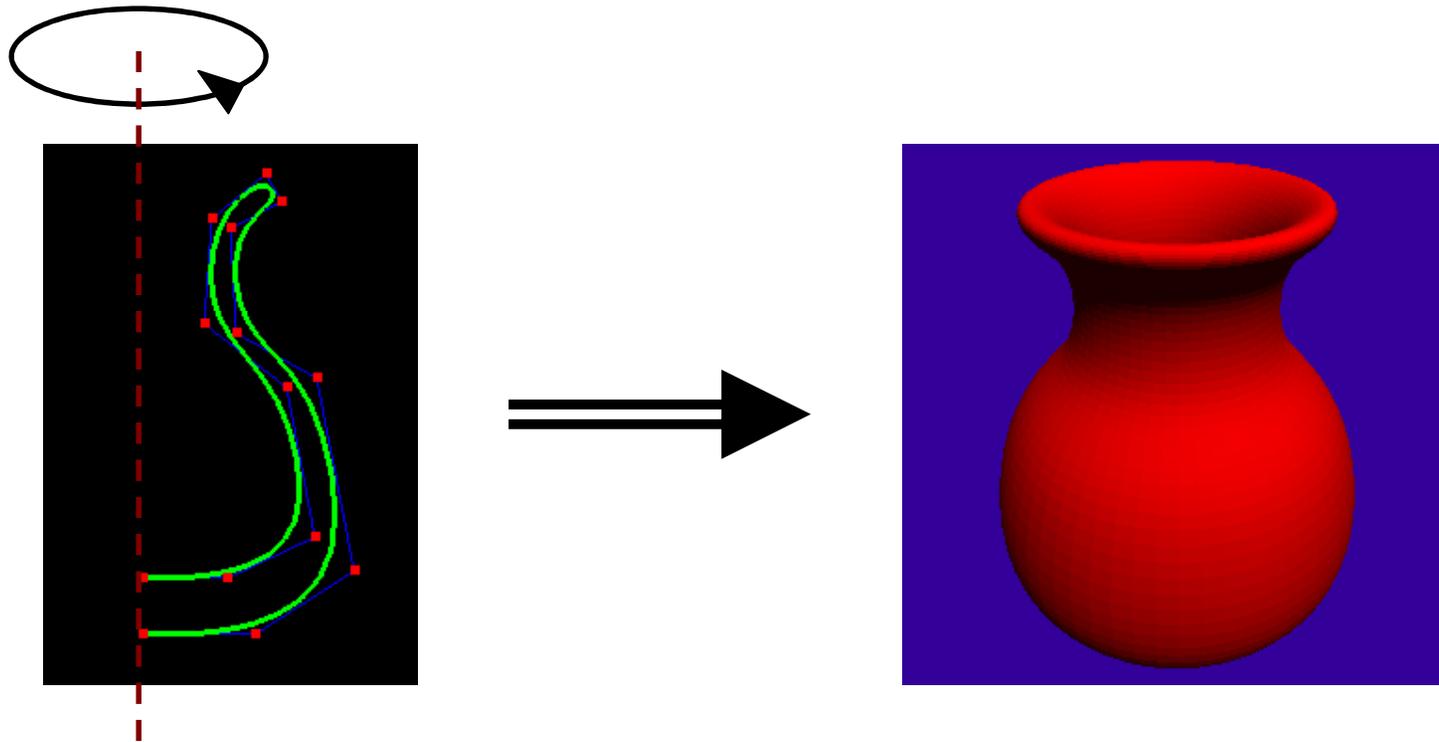
Modeling

- ▶ Creating 3D objects
- ▶ How to construct complex surfaces?
- ▶ Goal
 - ▶ Specify objects with control points
 - ▶ Objects should be visually pleasing (smooth)
- ▶ Start with curves, then generalize to surfaces
- ▶ Next: What can curves be used for?



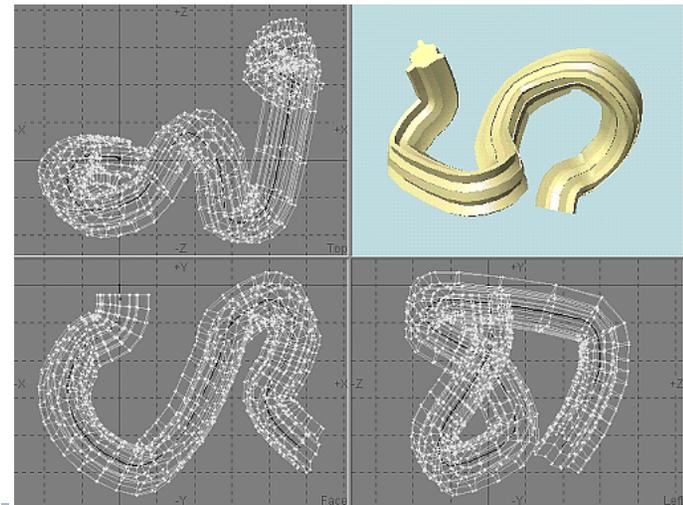
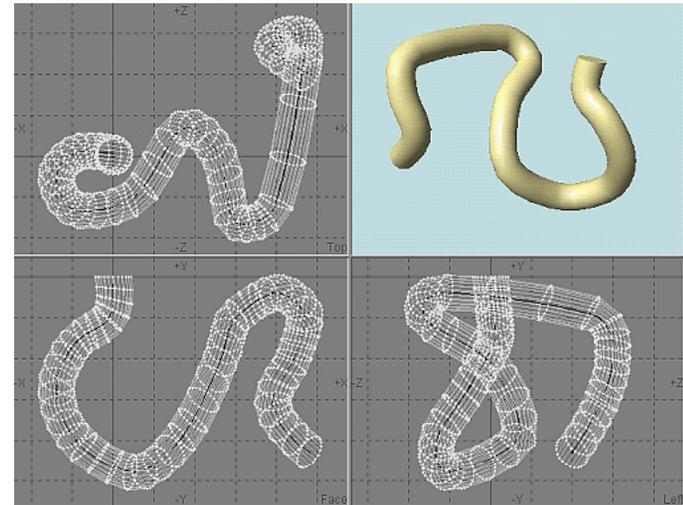
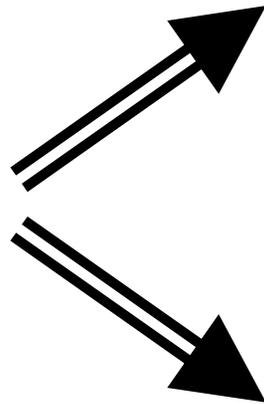
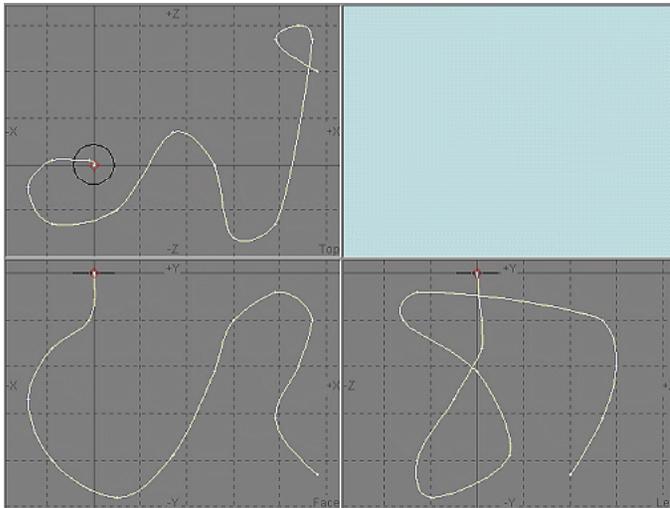
Curves

- ▶ Surface of revolution (homework project!)



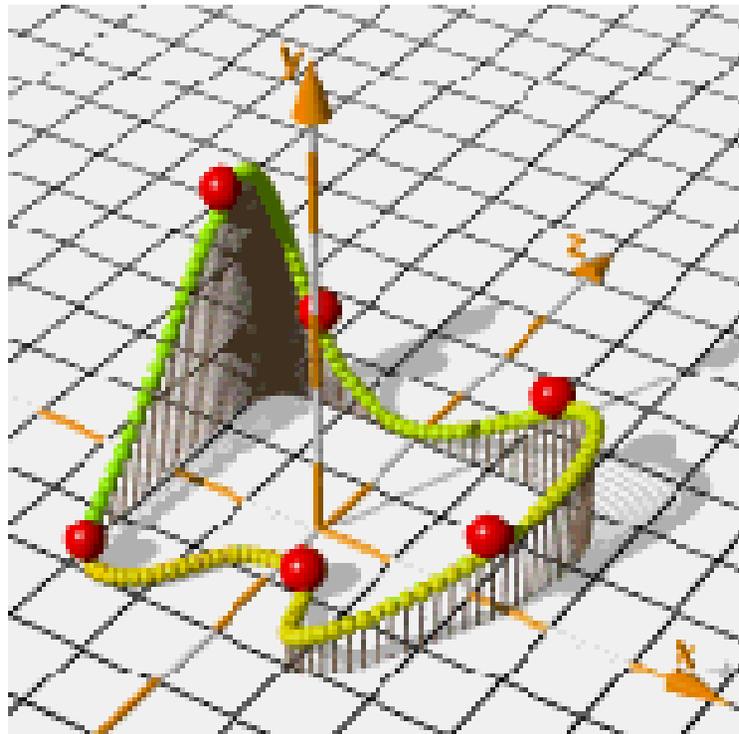
Curves

▶ Extruded/swept surfaces



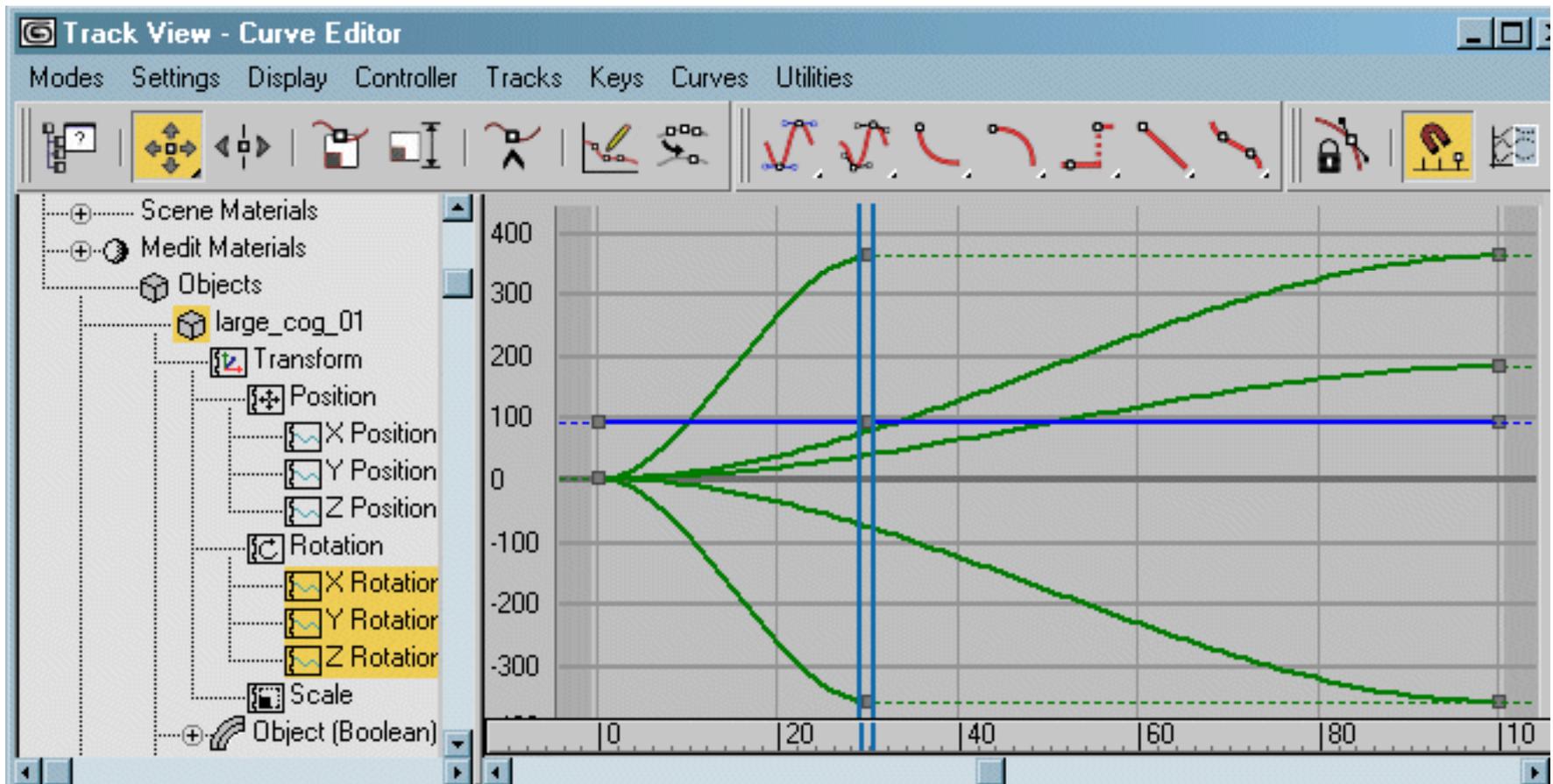
Curves

- ▶ **Animation**
 - ▶ Provide a “track” for objects
 - ▶ Use as camera path



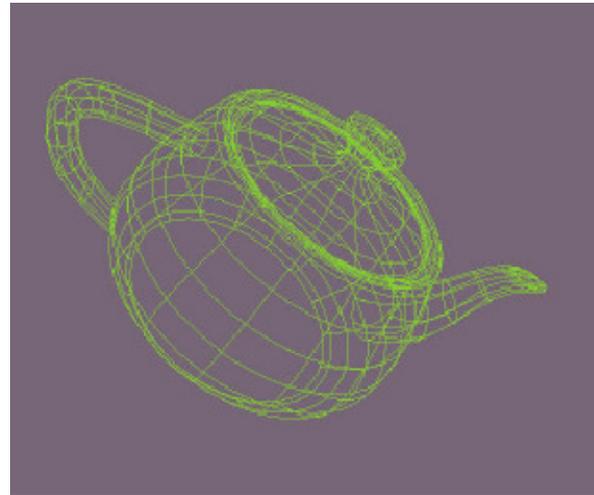
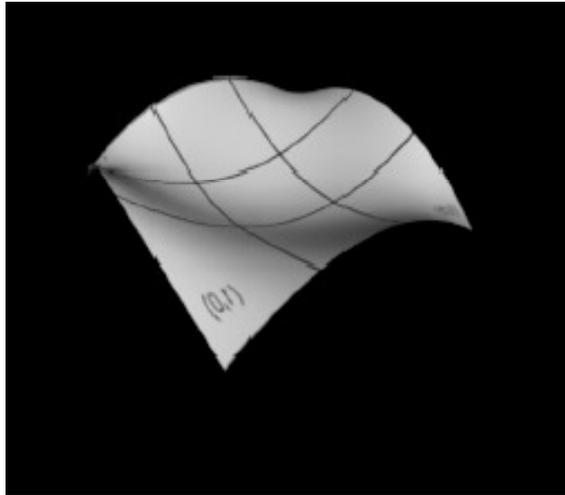
Curves

- ▶ Specify parameter values over time



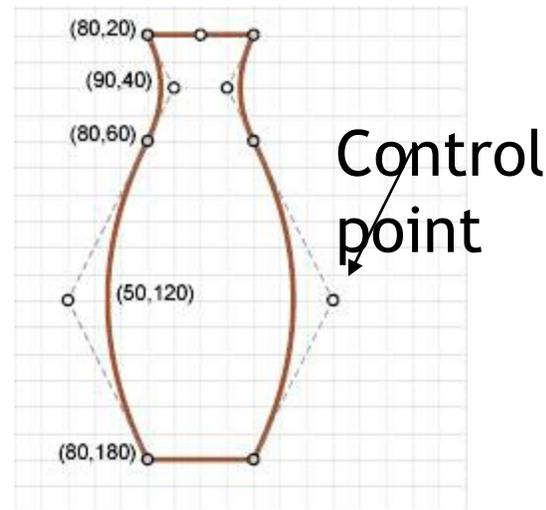
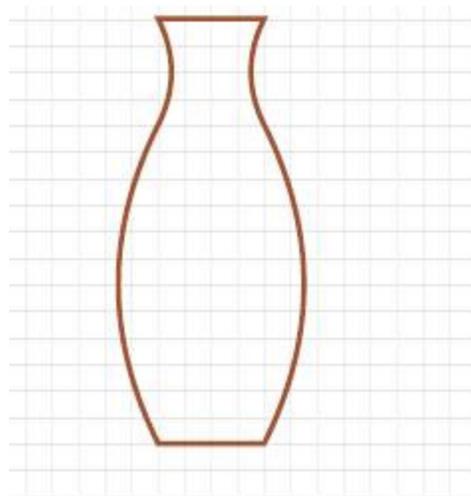
Curves

- ▶ Can be generalized to surface patches



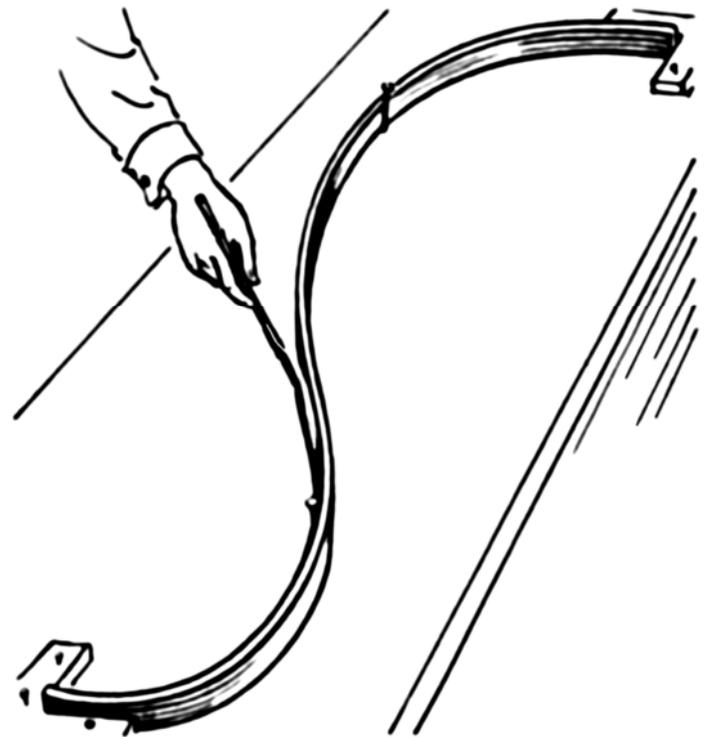
Curve Representation

- ▶ Specify every point along a curve?
 - ▶ Hard to get precise, smooth results
 - ▶ Too much data, difficult to work with
- ▶ Specify a curve using a small number of “control points”
 - ▶ Known as a *spline curve* or just *spline*



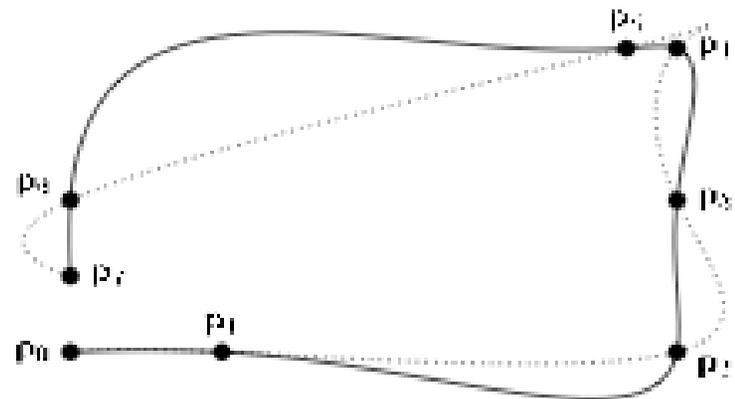
Spline: Definition

- ▶ **Wikipedia:**
 - ▶ Term comes from flexible spline devices used by shipbuilders and draftsmen to draw smooth shapes
 - ▶ Spline consists of a long strip fixed in position at a number of points that relaxes to form a smooth curve passing through those points



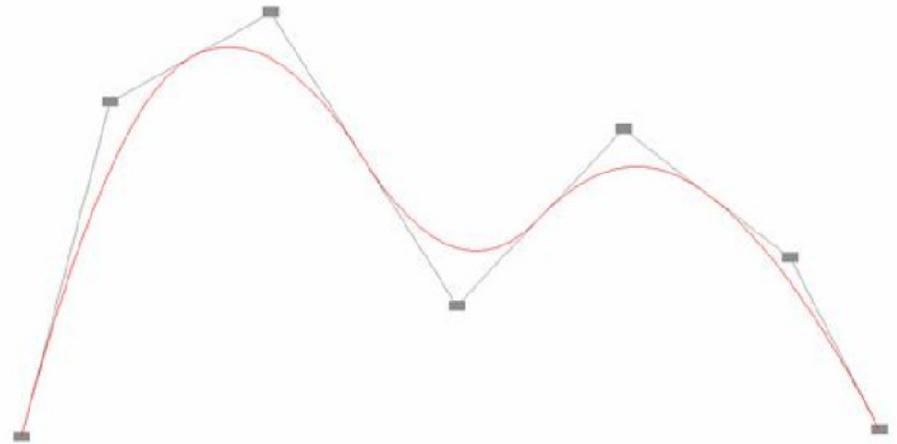
Interpolating Splines

- ▶ Curve goes through all control points
- ▶ Seems most intuitive
- ▶ Surprisingly, not usually the best choice
- ▶ Hard to predict behavior
 - ▶ Overshoots, wiggles
- ▶ Hard to get “nice-looking” curves



Approximating Splines

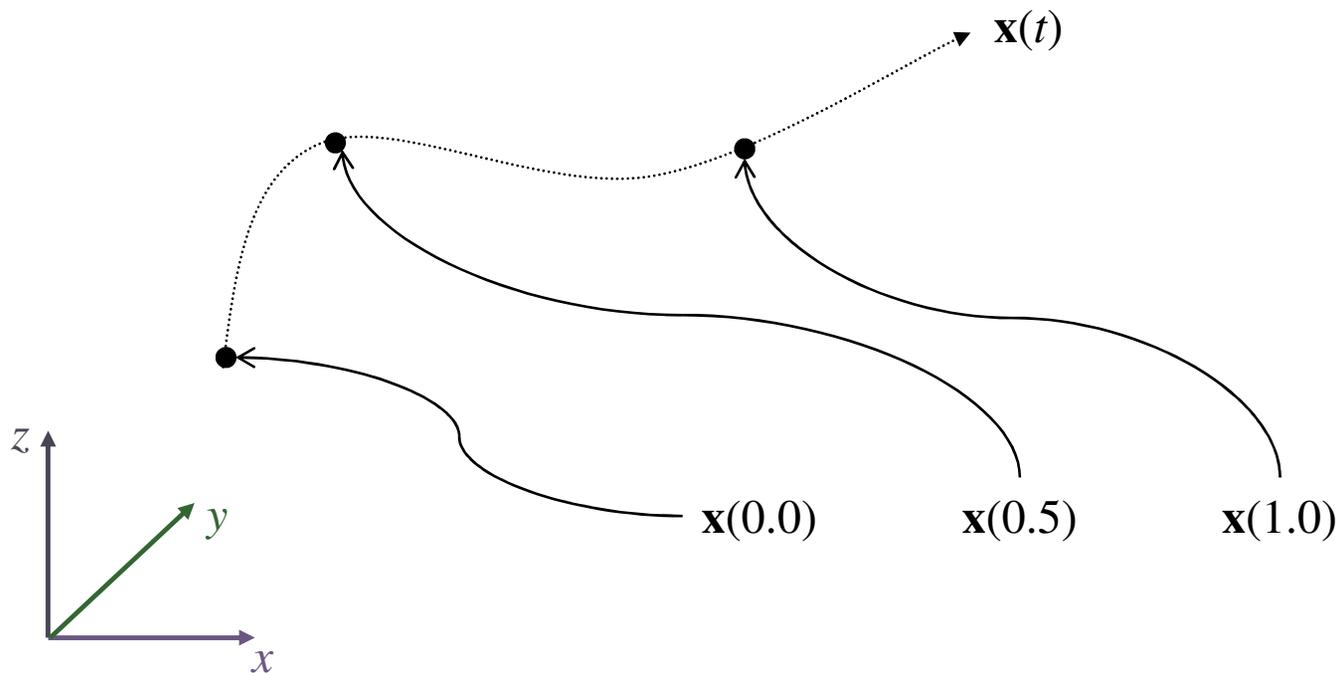
- ▶ Curve is “influenced” by control points



- ▶ Various types & techniques
- ▶ Most common: polynomial functions
 - ▶ Bézier spline
 - ▶ B-spline (generalization of Bézier spline)
 - ▶ NURBS (Non Uniform Rational Basis Spline)
- ▶ In this lecture: focus on Bézier splines

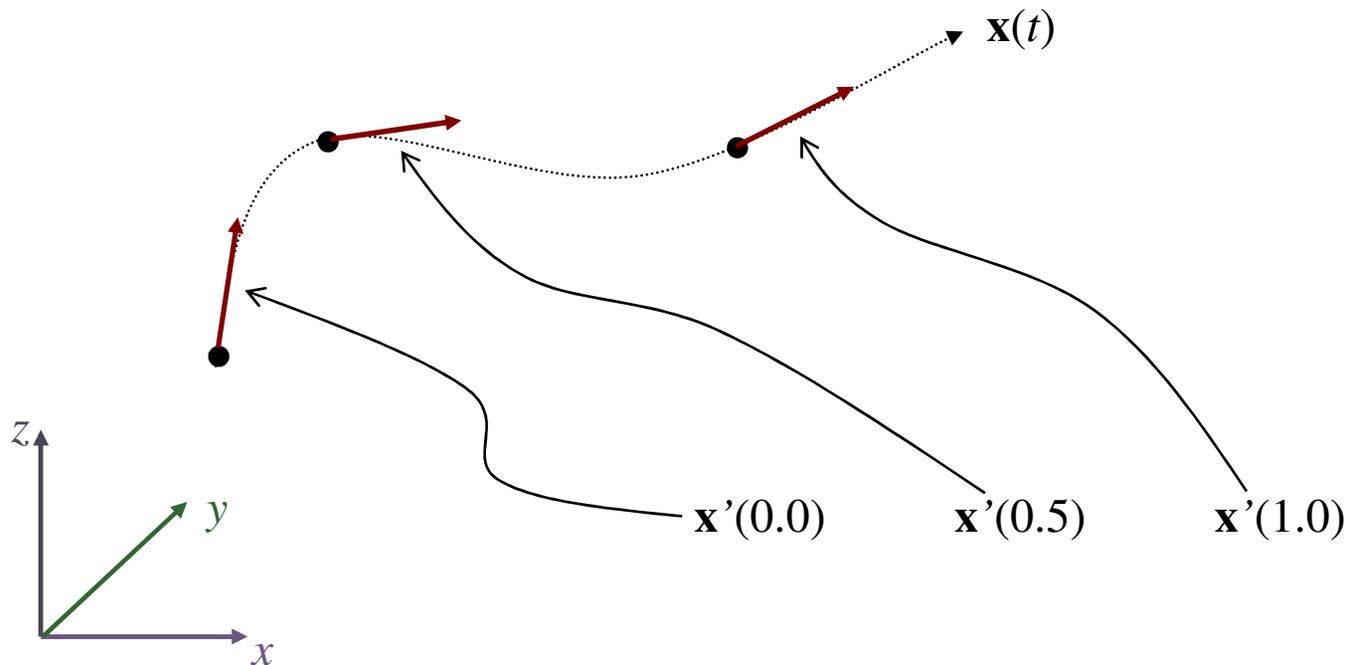
Mathematical Definition

- ▶ A vector valued function of one variable $\mathbf{x}(t)$
 - ▶ Given t , compute a 3D point $\mathbf{x}=(x,y,z)$
 - ▶ May interpret as three functions $x(t)$, $y(t)$, $z(t)$
 - ▶ “Moving a point along the curve”



Tangent Vector

- ▶ Derivative $\mathbf{x}'(t) = \frac{d\mathbf{x}}{dt} = (x'(t), y'(t), z'(t))$
- ▶ A vector that points in the direction of movement
- ▶ Length corresponds to speed

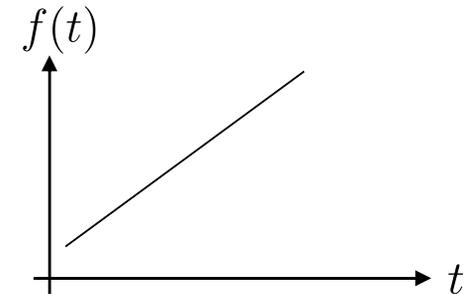


Lecture Overview

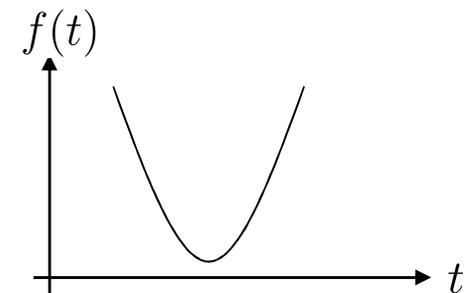
- ▶ **Scene Graphs & Hierarchies**
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Polynomial Functions

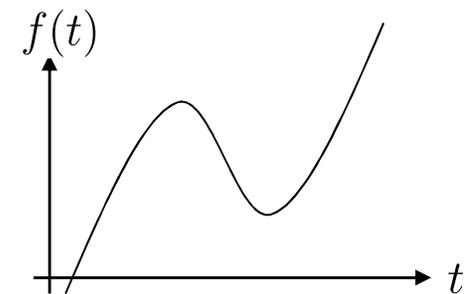
- ▶ **Linear:** $f(t) = at + b$
(1st order)



- ▶ **Quadratic:** $f(t) = at^2 + bt + c$
(2nd order)



- ▶ **Cubic:** $f(t) = at^3 + bt^2 + ct + d$
(3rd order)

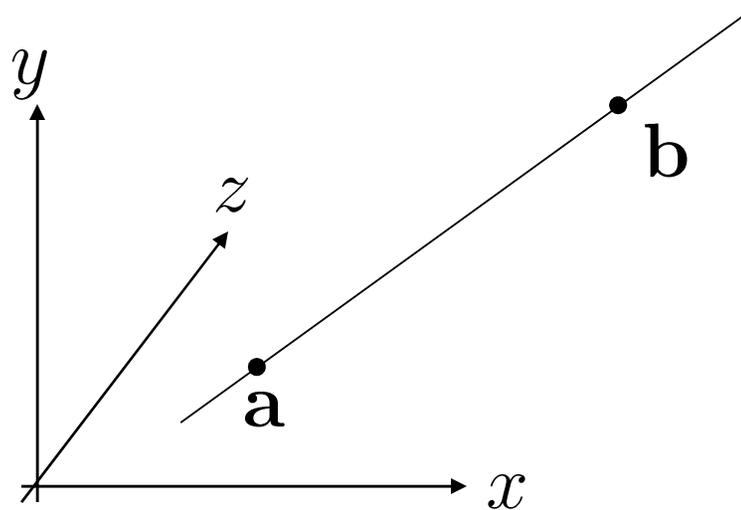


Polynomial Curves

- ▶ Linear $\mathbf{x}(t) = \mathbf{a}t + \mathbf{b}$

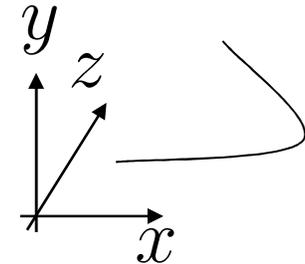
$$\mathbf{x} = (x, y, z), \mathbf{a} = (a_x, a_y, a_z), \mathbf{b} = (b_x, b_y, b_z)$$

- ▶ Evaluated as:
 $x(t) = a_x t + b_x$
 $y(t) = a_y t + b_y$
 $z(t) = a_z t + b_z$

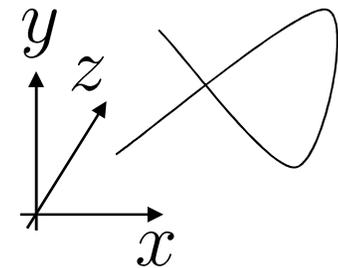


Polynomial Curves

▶ **Quadratic:** $\mathbf{x}(t) = \mathbf{a}t^2 + \mathbf{b}t + \mathbf{c}$
(2nd order)



▶ **Cubic:** $\mathbf{x}(t) = \mathbf{a}t^3 + \mathbf{b}t^2 + \mathbf{c}t + \mathbf{d}$
(3rd order)



▶ We usually define the curve for $0 \leq t \leq 1$

Control Points

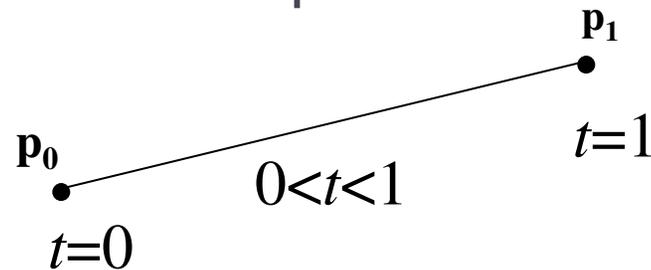
- ▶ Polynomial coefficients **a**, **b**, **c**, **d** can be interpreted as *control points*
 - ▶ Remember: **a**, **b**, **c**, **d** have x, y, z components each
- ▶ Unfortunately, they don't intuitively describe the shape of the curve
- ▶ Main objective of curve representation is to come up with intuitive control points

Control Points

- ▶ **How many control points?**
 - ▶ Two points define a line (1st order)
 - ▶ Three points define a quadratic curve (2nd order)
 - ▶ Four points define a cubic curve (3rd order)
 - ▶ $k+1$ points define a k -order curve
- ▶ **Let's start with a line...**

First Order Curve

- ▶ Based on linear interpolation (LERP)
 - ▶ Weighted average between two values
 - ▶ “Value” could be a number, vector, color, ...
- ▶ Interpolate between points \mathbf{p}_0 and \mathbf{p}_1 with parameter t
 - ▶ Defines a “curve” that is straight (first-order spline)
 - ▶ $t=0$ corresponds to \mathbf{p}_0
 - ▶ $t=1$ corresponds to \mathbf{p}_1
 - ▶ $t=0.5$ corresponds to midpoint



$$\mathbf{x}(t) = \text{Lerp}(t, \mathbf{p}_0, \mathbf{p}_1) = (1 - t)\mathbf{p}_0 + t \mathbf{p}_1$$

Linear Interpolation

- ▶ Three different ways to write it

- ▶ All equivalent

- ▶ Different properties become apparent

1. Weighted sum of the control points

$$\mathbf{x}(t) = \mathbf{p}_0(1 - t) + \mathbf{p}_1t$$

2. Polynomial in t

$$\mathbf{x}(t) = (\mathbf{p}_1 - \mathbf{p}_0)t + \mathbf{p}_0$$

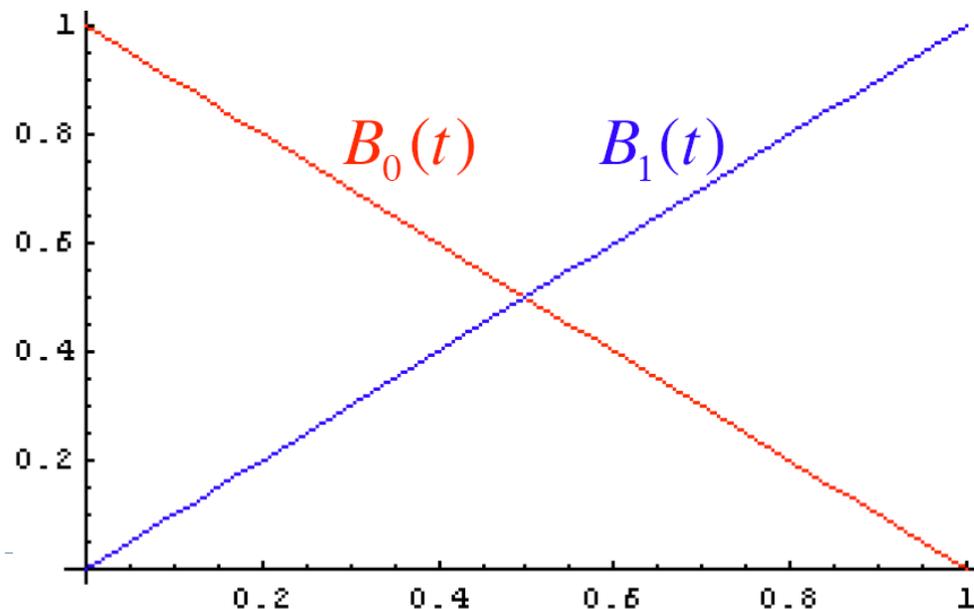
3. Matrix form

$$\mathbf{x}(t) = \begin{bmatrix} \mathbf{p}_0 & \mathbf{p}_1 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} t \\ 1 \end{bmatrix}$$

Weighted Average

$$\begin{aligned}\mathbf{x}(t) &= (1-t)\mathbf{p}_0 + t\mathbf{p}_1 \\ &= B_0(t)\mathbf{p}_0 + B_1(t)\mathbf{p}_1, \text{ where } B_0(t) = 1-t \text{ and } B_1(t) = t\end{aligned}$$

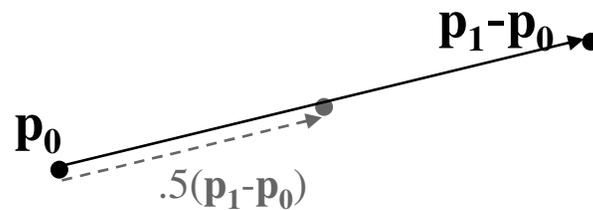
- ▶ Weights are a function of t
 - ▶ Sum is always 1, for any value of t
 - ▶ Also known as *blending functions*



Linear Polynomial

$$\mathbf{x}(t) = \underbrace{(\mathbf{p}_1 - \mathbf{p}_0)}_{\text{vector } \mathbf{a}} t + \underbrace{\mathbf{p}_0}_{\text{point } \mathbf{b}}$$

- ▶ Curve is based at point \mathbf{p}_0
- ▶ Add the vector, scaled by t



Matrix Form

$$\mathbf{x}(t) = \begin{bmatrix} \mathbf{p}_0 & \mathbf{p}_1 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} t \\ 1 \end{bmatrix} = \mathbf{GBT}$$

▶ Geometry matrix $\mathbf{G} = \begin{bmatrix} \mathbf{p}_0 & \mathbf{p}_1 \end{bmatrix}$

▶ Geometric basis $\mathbf{B} = \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix}$

▶ Polynomial basis $T = \begin{bmatrix} t \\ 1 \end{bmatrix}$

▶ In components $\mathbf{x}(t) = \begin{bmatrix} p_{0x} & p_{1x} \\ p_{0y} & p_{1y} \\ p_{0z} & p_{1z} \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} t \\ 1 \end{bmatrix}$

Tangent

- ▶ For a straight line, the tangent is constant

$$\mathbf{x}'(t) = \mathbf{p}_1 - \mathbf{p}_0$$

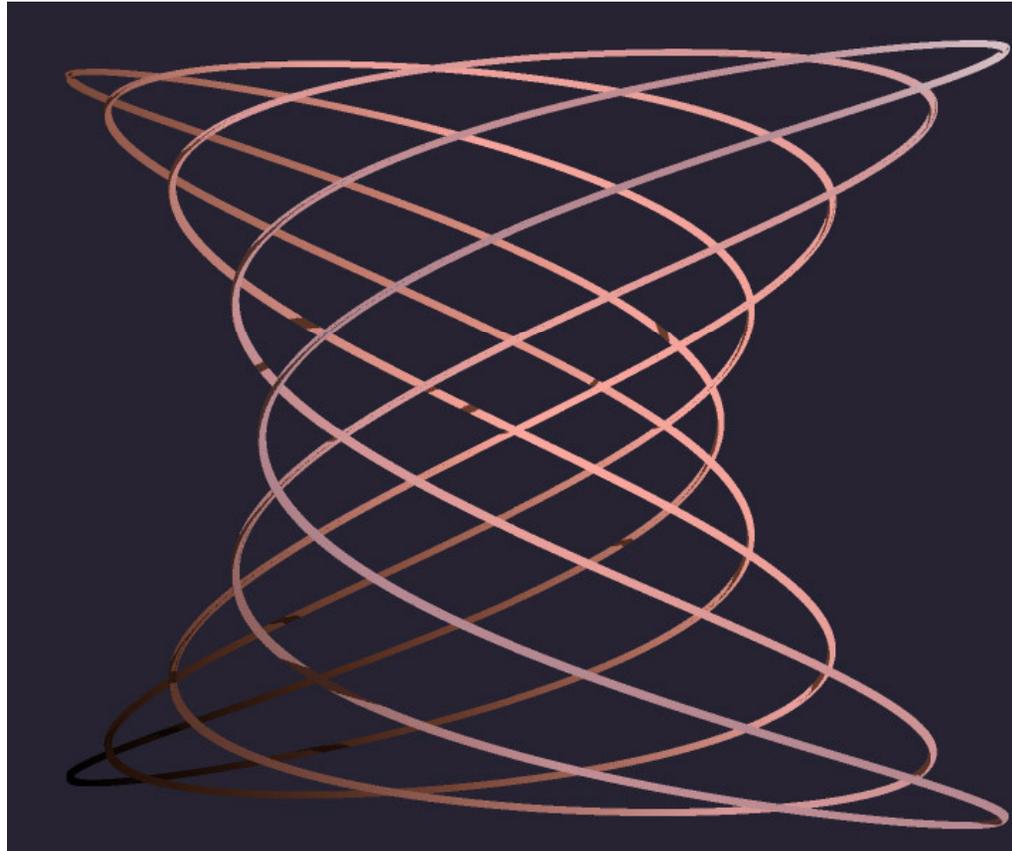
- ▶ Weighted average $\mathbf{x}'(t) = (-1)\mathbf{p}_0 + (+1)\mathbf{p}_1$

- ▶ Polynomial $\mathbf{x}'(t) = 0t + (\mathbf{p}_1 - \mathbf{p}_0)$

- ▶ Matrix form $\mathbf{x}'(t) = \begin{bmatrix} \mathbf{p}_0 & \mathbf{p}_1 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

Lissajous Curves

- ▶ Live demo: <http://ibiblio.org/e-notes/Lis/Lissa.htm>



http://en.wikipedia.org/wiki/Lissajous_curve

Next Lecture

- ▶ Bezier curves
- ▶ Curves with multiple segments
- ▶ Extension to surfaces