CSE 167:
Introduction to Computer Graphics
Lecture #4: Rasterization

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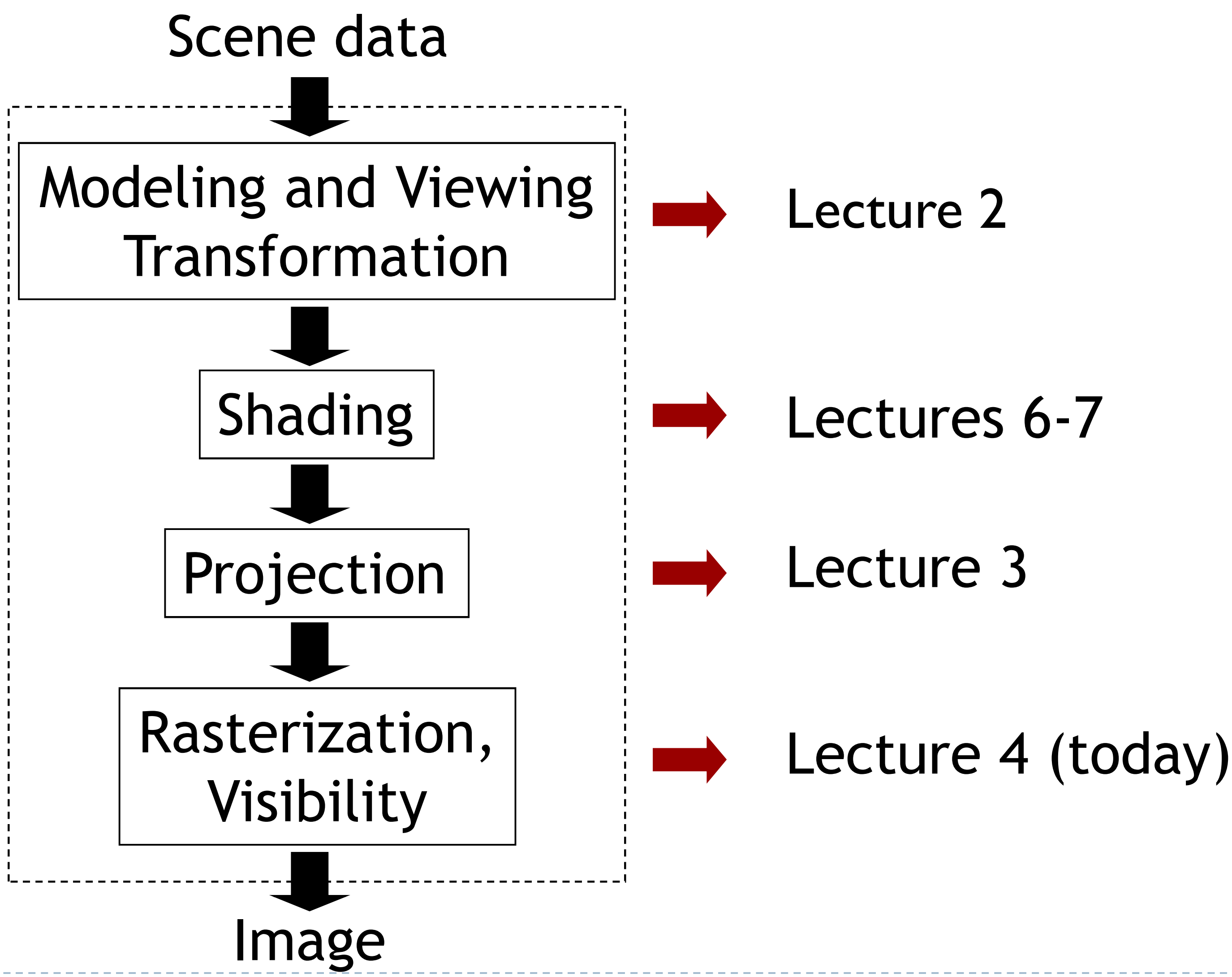
Announcements

- Project 2 due Friday, October 8. Grading slot is now 2-4pm.
- Gradesource IDs should have been sent to everybody's email addresses. Check Gradesource for scores. Scores may not be available until after late grading deadline.
- I need PID and email addresses from the following students, to give Gradesource access. Please see me after class, or send me email.
  - Kuttuva, Manjuladevi
  - Li, Yunjiu
  - Lorenz-Meyer, Vitus
  - Viswanathan, Ramesh
  - Ramos, Facundo

Lecture Overview

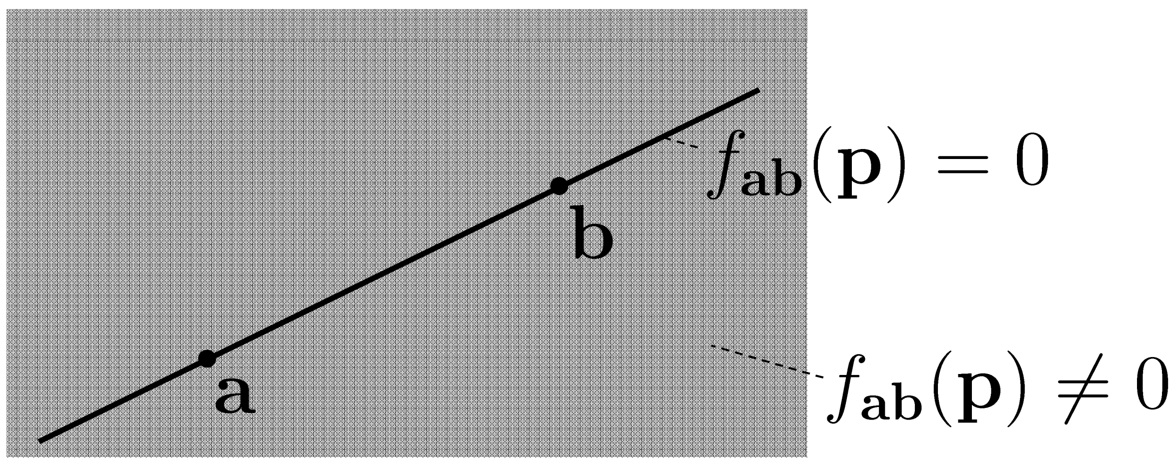
- Barycentric Coordinates
- Culling, Clipping
- Rasterization
- Visibility

Rendering Pipeline



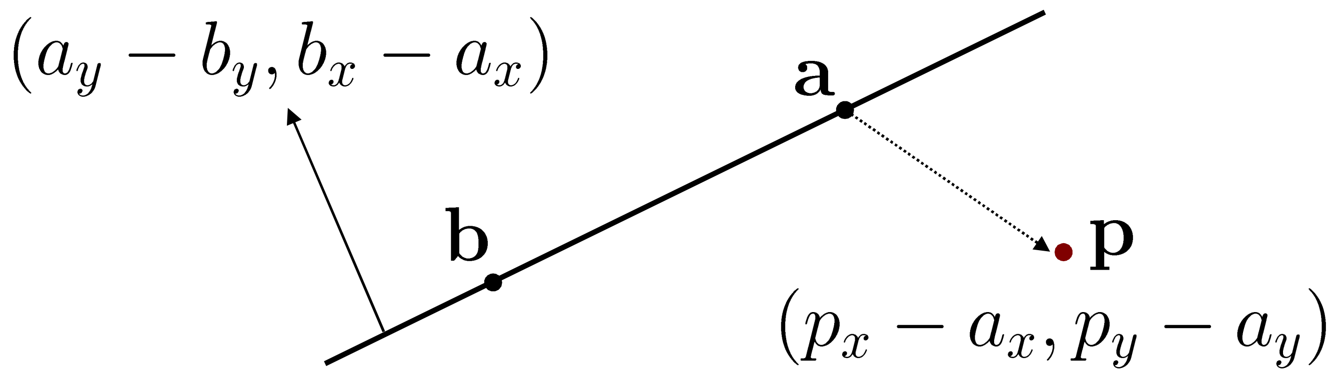
Implicit 2D Lines

- Given two 2D points **a**, **b**
- Define function $f_{\mathbf{ab}}(\mathbf{p})$ such that $f_{\mathbf{ab}}(\mathbf{p}) = 0$ if **p** lies on the line defined by **a**, **b**



Implicit 2D Lines

- Point **p** lies on the line, if **p-a** is perpendicular to the normal of the line

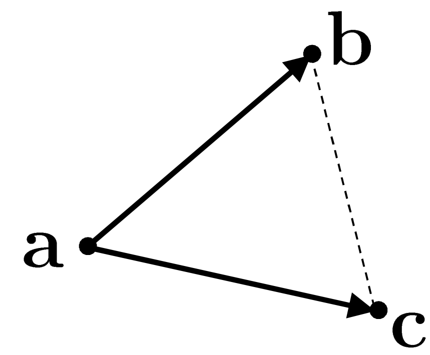


- Use dot product to determine on which side of the line **p** lies. If f(**p**)>0, **p** is on same side as normal, if f(**p**)<0 **p** is on opposite side. If dot product is 0, **p** lies on the line.

$$f_{\mathbf{ab}}(\mathbf{p}) = (a_y - b_y, b_x - a_x) \cdot (p_x - a_x, p_y - a_y)$$

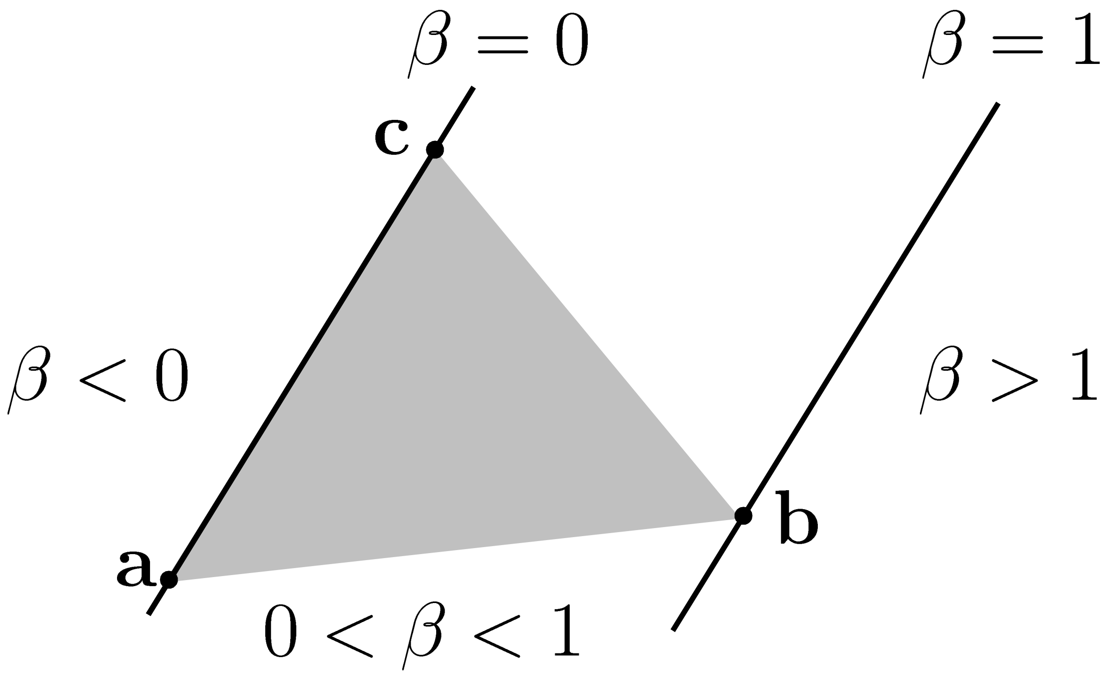
Barycentric Coordinates

- Coordinates for 2D plane defined by triangle vertices ***a, b, c***
- Any point ***p*** in the plane defined by ***a, b, c*** is
  $\boldsymbol{p} = a + \beta(b - a) + \gamma(c - a)$
  $= (1 - \beta - \gamma)\, a + \beta b + \gamma c$
- We define $\alpha = 1 - \beta - \gamma$
  $\Rightarrow \boldsymbol{p} = \alpha a + \beta b + \gamma c$
- α, β, γ are called **barycentric** coordinates
- Works in 2D and in 3D
- If we imagine masses equal to α, β, γ attached to the vertices of the triangle, the center of mass (the barycenter) is then ***p***. This is the origin of the term “barycentric” (introduced 1827 by Möbius)



Barycentric Coordinates

$$\mathbf{p} \;=\; \mathbf{a} + \beta(\mathbf{b} - \mathbf{a}) + \gamma(\mathbf{c} - \mathbf{a})$$



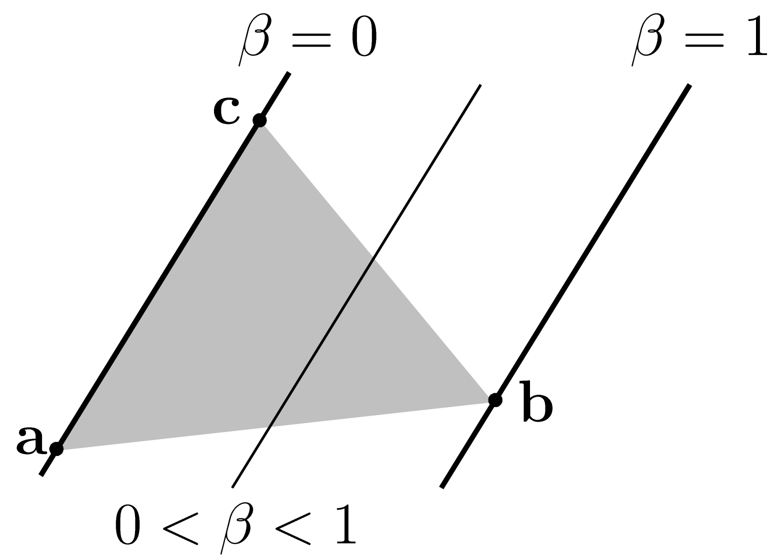
- ***p*** is inside the triangle if $0 < \alpha, \beta, \gamma < 1$

Barycentric Coordinates

- Problem: Given point ***p***, find its barycentric coordinates
- Use equation for implicit lines

$$\beta(\mathbf{p}) = \frac{f_{\mathbf{ac}}(\mathbf{p})}{f_{\mathbf{ac}}(\mathbf{b})}$$

$$\gamma(\mathbf{p}) = \frac{f_{\mathbf{ab}}(\mathbf{p})}{f_{\mathbf{ab}}(\mathbf{c})}$$



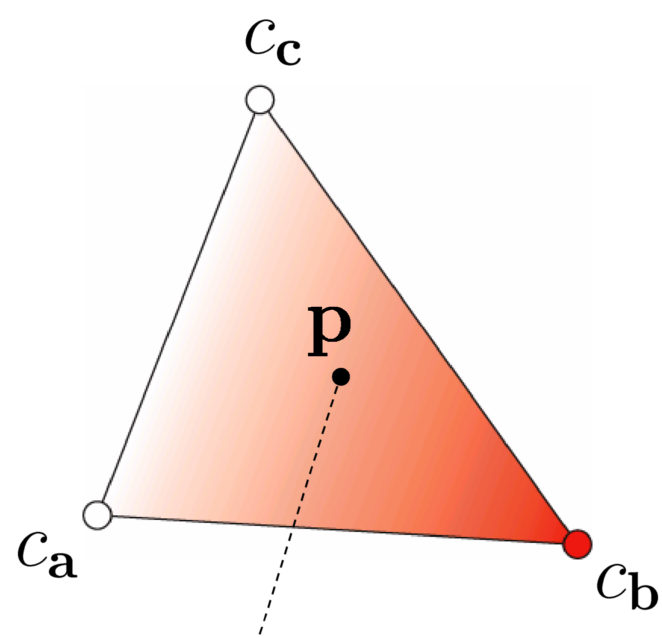
- Division by zero if triangle is degenerate

$$\alpha = 1 - \beta - \gamma$$

$$0 < \beta < 1$$

Barycentric Interpolation

- Interpolate values across triangles, e.g., colors



- Linear interpolation on triangles

$$c(\mathbf{p}) = \alpha(\mathbf{p})c_{\mathbf{a}} + \beta(\mathbf{p})c_{\mathbf{b}} + \gamma(\mathbf{p})c_{\mathbf{c}}$$

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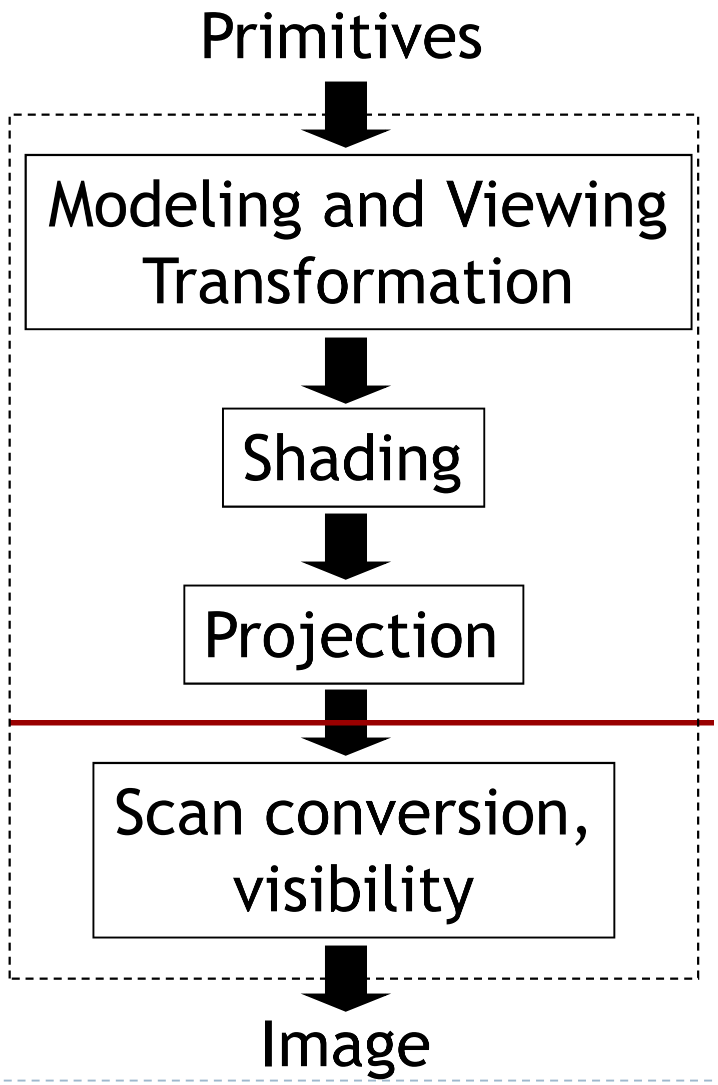
Rendering Pipeline



Culling, Clipping

- Discard geometry that should not be drawn

Culling

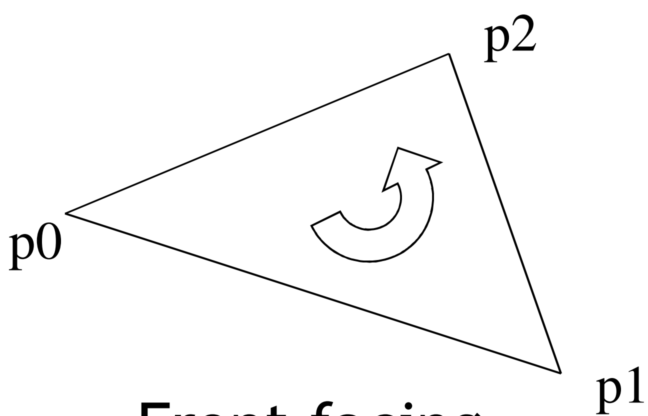
- Discard geometry that does not need to be drawn as early as possible
- Two types of culling:
  - Object-level frustum culling
    - Later in class
  - Triangle culling
    - View frustum culling (clipping): outside view frustum
    - Backface culling: facing "away" from the viewer
    - Degenerate culling: area=0

Backface Culling

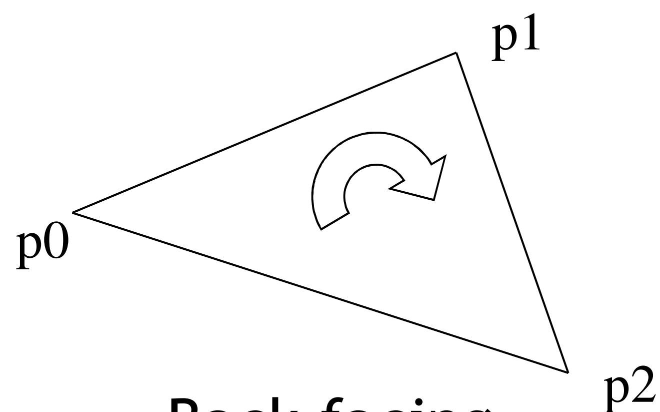
- Consider triangles as "one-sided", i.e., only visible from the "front"
- Closed objects
  - If the "back" of the triangle is facing the camera, it is not visible
  - Gain efficiency by not drawing it (culling)
  - Roughly 50% of triangles in a scene are back facing

Backface Culling

- Convention: front side means vertices are ordered counterclockwise



Front facing



Back facing

- OpenGL allows one- or two-sided triangles
  - One-sided triangles: glEnable(GL_CULL_FACE); glCullFace(GL_BACK)
  - Two-sided triangles (no backface culling): glDisable(GL_CULL_FACE)

Backface Culling

- Compute triangle normal after projection (homogeneous division)

$$\mathbf{n} = (\mathbf{p}_1 - \mathbf{p}_0) \times (\mathbf{p}_2 - \mathbf{p}_0)$$

- Third component of $\mathbf{n}$ negative: front-facing, otherwise back-facing
  - Remember: projection matrix is such that homogeneous division flips sign of third component

Degenerate Culling

- Degenerate triangle has no area
  - Vertices lie in a straight line
  - Vertices at the exact same place
  - Normal $\mathbf{n}=0$

View Frustum Culling, Clipping

- Triangles that intersect the faces of the view volume
  - Partly on screen, partly off screen
  - Do not rasterize the parts that are off-screen
- Traditional clipping
  - Split triangles that lie partly inside/outside viewing volume before homogeneous division
  - Avoid problems with division by zero
- Modern GPU implementations avoid clipping

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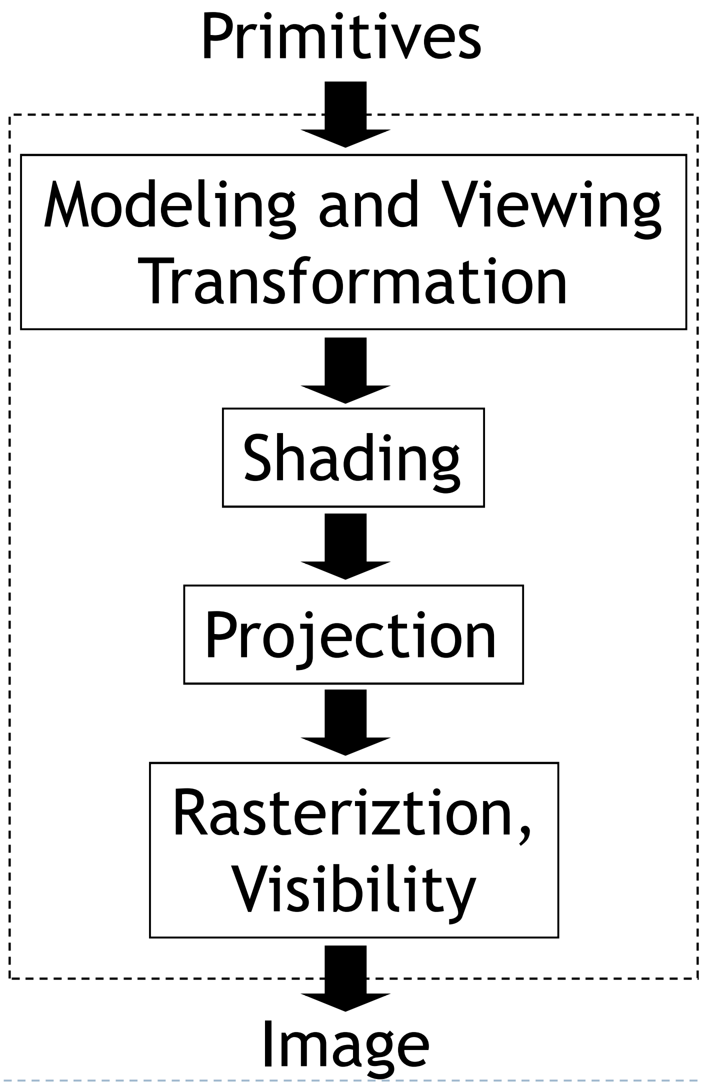
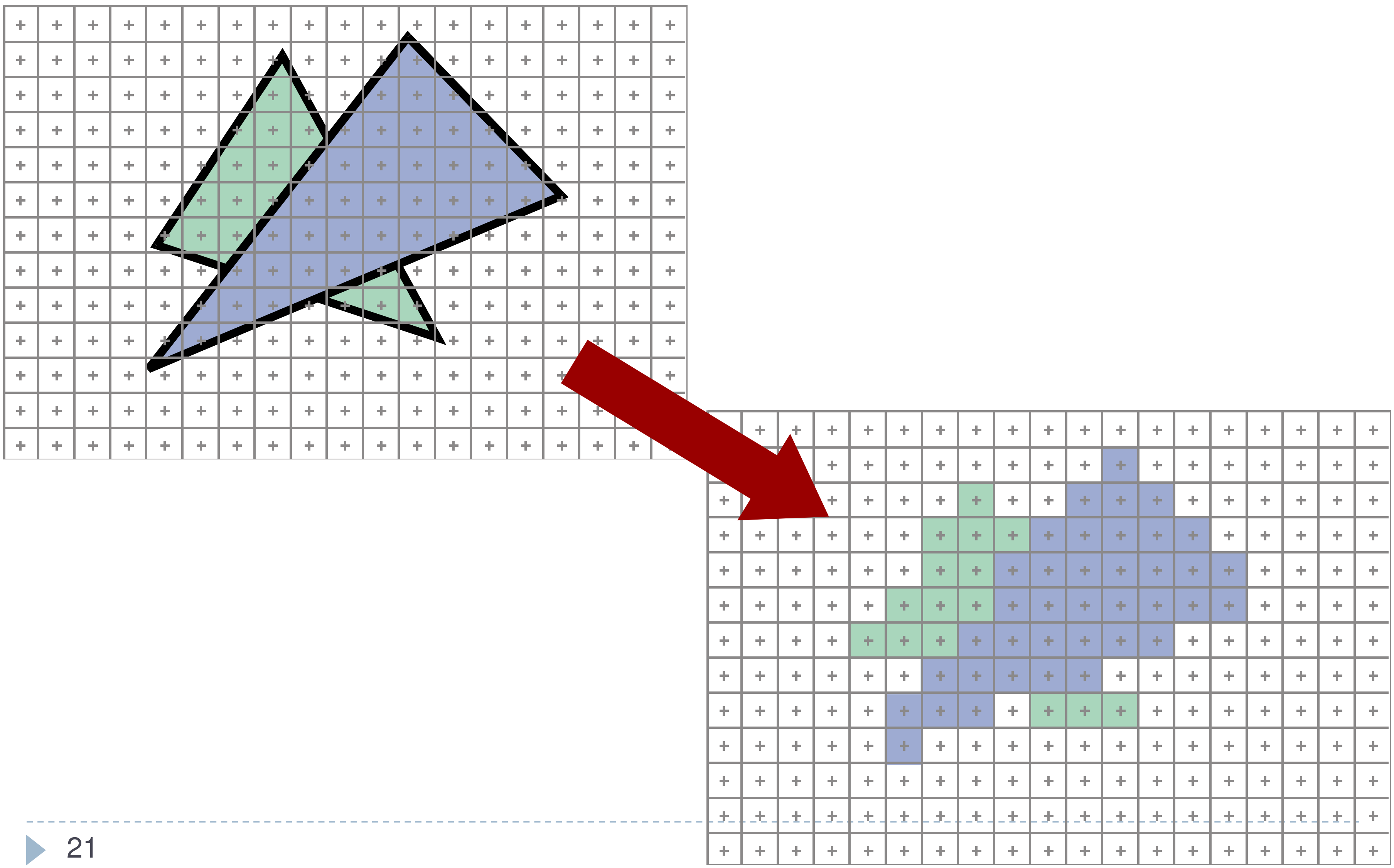
Rendering Pipeline



- Scan conversion and rasterization are synonyms
- One of the main operations performed by GPU
- Draw triangles, lines, points (squares)
- Focus on triangles in this lecture

Rasterization



Rasterization

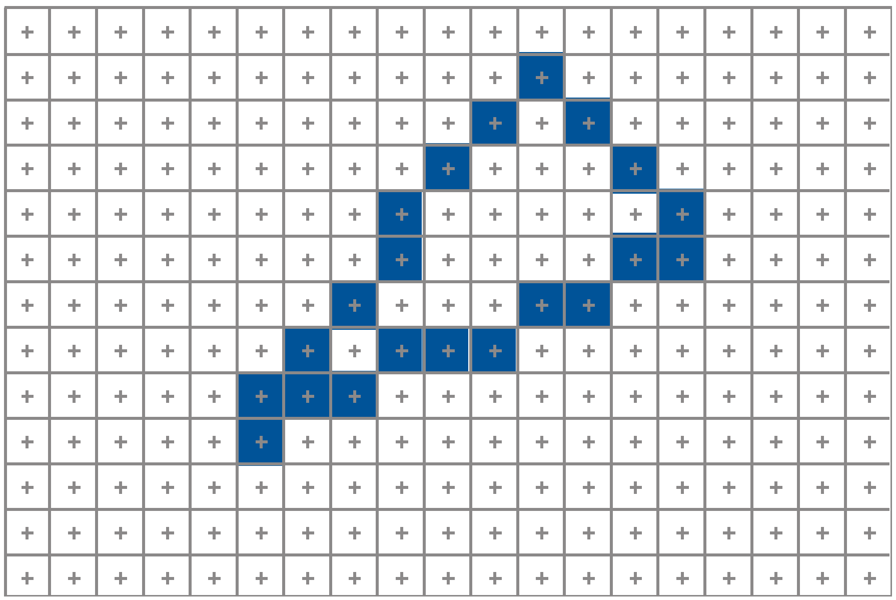
- How many pixels can a modern graphics processor draw per second?

Rasterization

- Rasterization is „hard-coded“ in the graphics card, cannot (currently) be modified by the software
- NVidia GeForce 480 GTX
  - 33.6 billion pixels per second (GPix/s)
  - Multiple of what the fastest CPU could do

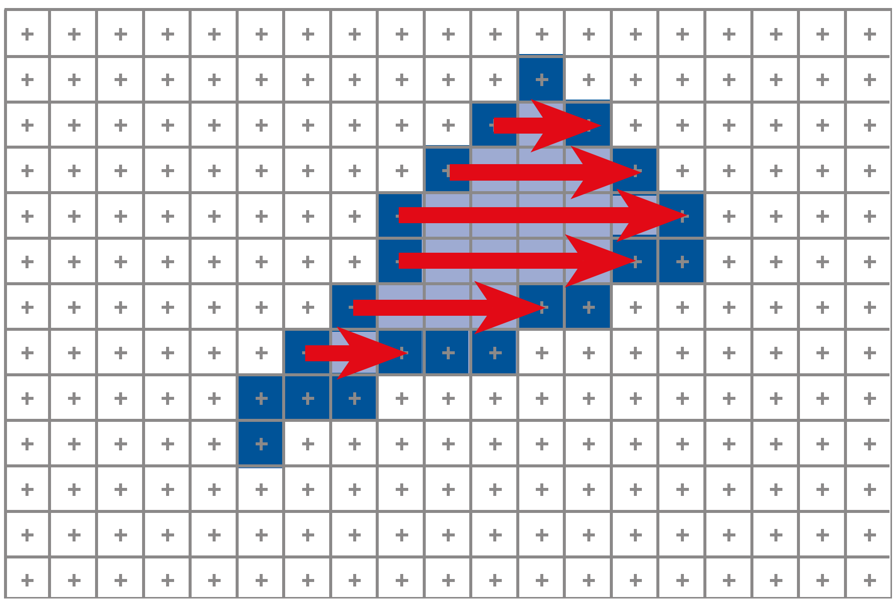
Rasterization

- Many different algorithms
- Old style
  - Rasterize edges first



Rasterization

- Many different algorithms
- Old style
  - Rasterize edges first
  - Fill the spans (scan lines, scan conversion)



Rasterization

- Many different algorithms exist
- Old style
  - Rasterize edges first
  - Fill the spans (scan lines, scan conversion)
  - Requires clipping
  - Straightforward, but not used for hardware implementation today

Rasterization

- GPU rasteriazation today based on "Homogeneous Rasterization"

  http://www.ece.unm.edu/course/ece595/docs/olano.pdf

  Olano, Marc and Trey Greer, "Triangle Scan Conversion Using 2D Homogeneous Coordinates", Proceedings of the 1997 SIGGRAPH/Eurographics Workshop on Graphics Hardware (Los Angeles, CA, August 2-4, 1997), ACM SIGGRAPH, New York, 1995.
- Does not require full clipping, does not perform homogeneous division at vertices
- Today in class
  - Simpler algorithm based on barycentric coordinates
  - More sophisticated than old style algorithm
  - Easy to implement
  - Requires clipping

Rasterization

- Given vertices in pixel coordinates

$$\mathbf{p}' = \mathbf{D}\mathbf{P}\mathbf{C}^{-1}\mathbf{M}\mathbf{p}$$

World space

Camera space

Clip space

Image space

$$\mathbf{p}' = \begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix}$$

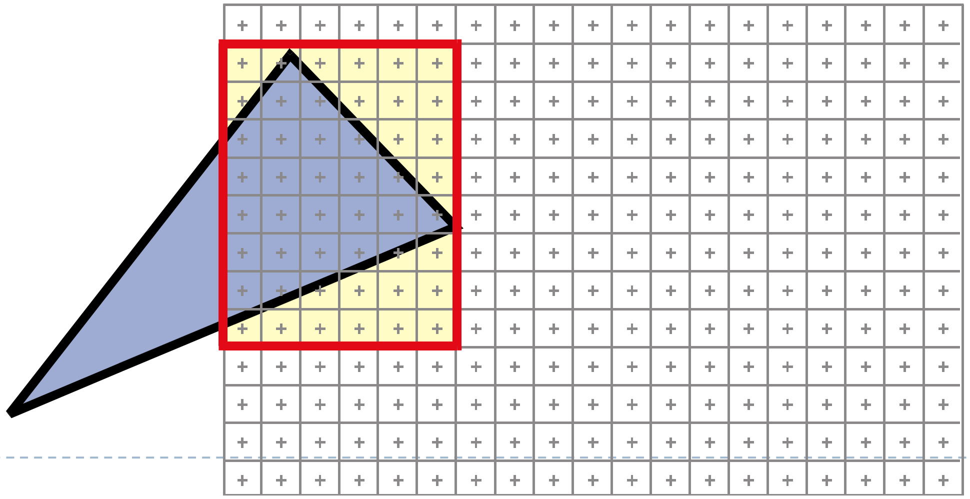
Pixel coordinates $\begin{matrix} x'/w' \\ y'/w' \end{matrix}$

Rasterization

- Simple algorithm

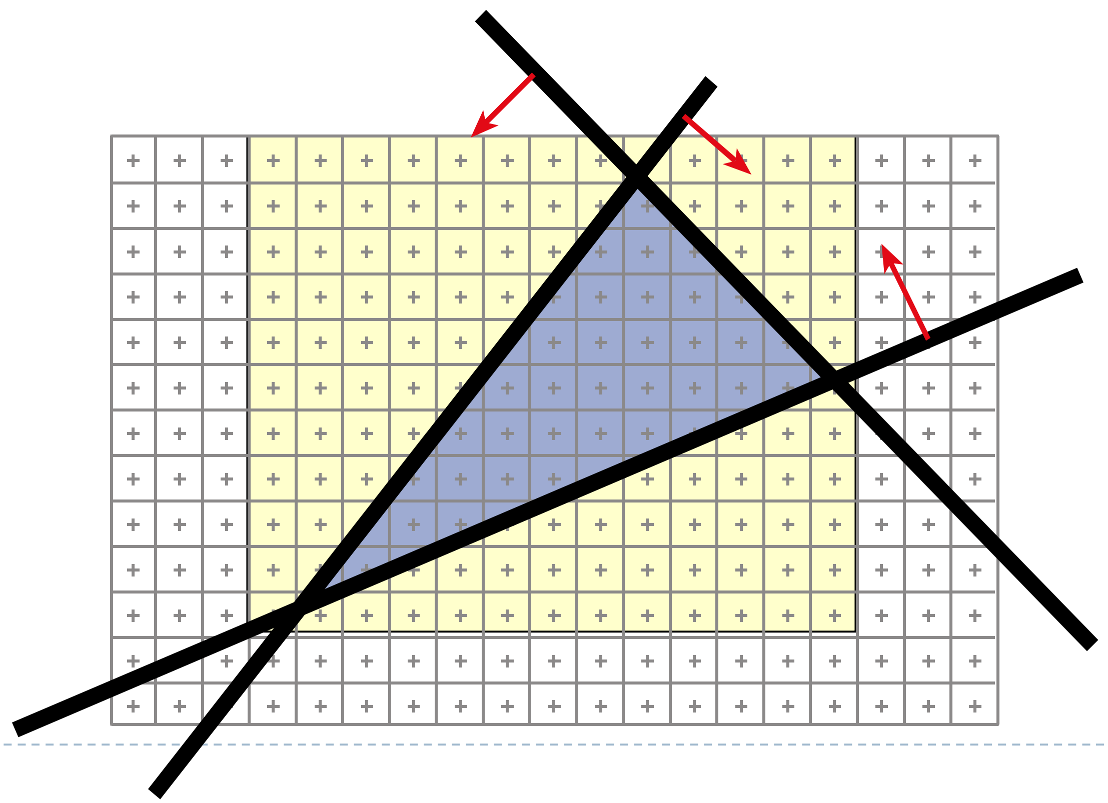
```
compute bbox
clip bbox to screen limits
for all pixels [x,y] in bbox
   compute barycentric coordinates alpha, beta, gamma
   if 0<alpha,beta,gamma<1 //pixel in triangle
            image[x,y]=triangleColor
```

- Bounding box clipping trivial



Rasterization

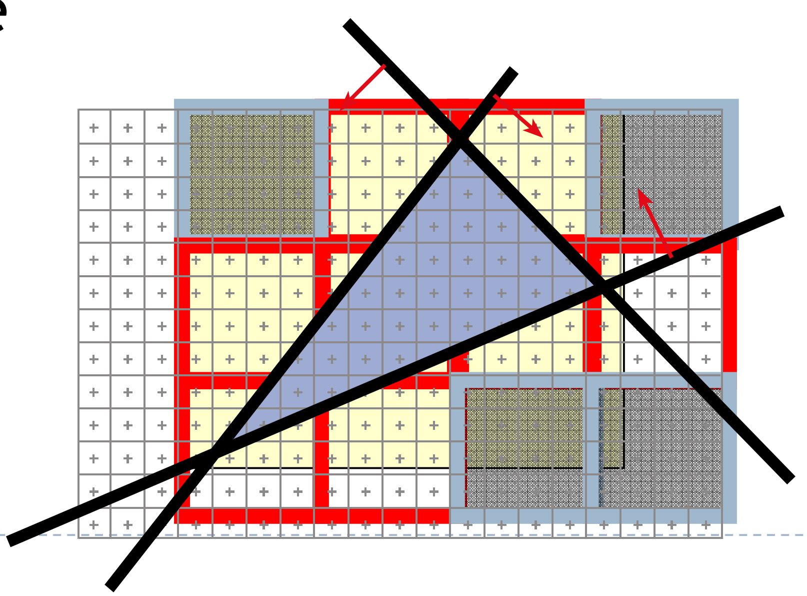
- So far, we compute barycentric coordinates of many useless pixels
- How can this be improved?



Rasterization

Hierarchy

- If block of pixels is outside triangle, no need to test individual pixels
- Can have several levels, usually two-level
- Find right granularity and size of blocks for optimal performance



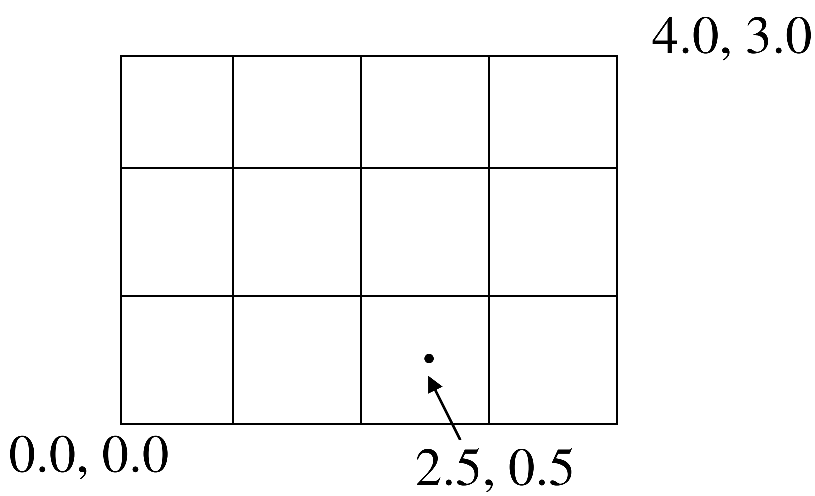
2D Triangle-Rectangle Intersection

- If one of the following tests returns true, the triangle intersects the rectangle:
  - Test if any of the triangle's vertices are inside the rectangle (e.g., by comparing the x/y coordinates to the min/max x/y coordinates of the rectangle)
  - Test if one of the quad's vertices is inside the triangle (e.g., using barycentric coordinates)
  - Intersect all edges of the triangle with all edges of the rectangle

Rasterization

Where is the center of a pixel?

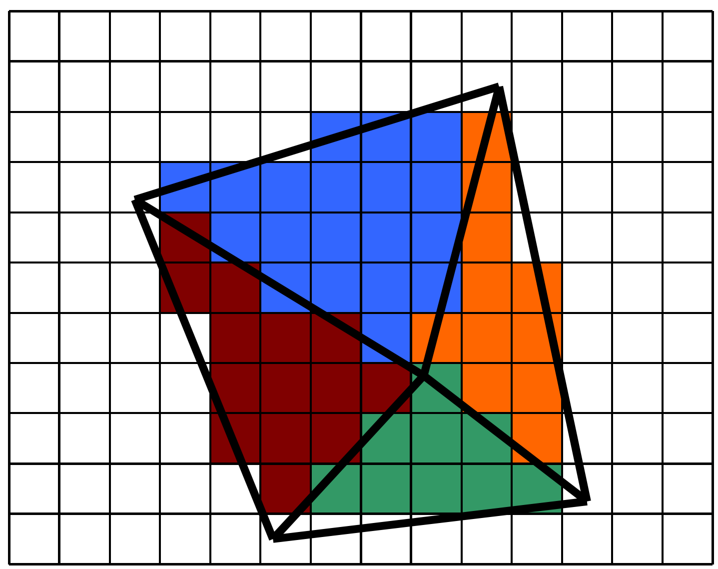
- Depends on conventions
- With our viewport transformation:
  - 800 x 600 pixels ⇔ viewport coordinates are in [0…800]x[0…600]
  - Center of lower left pixel is 0.5, 0.5
  - Center of upper right pixel is 799.5, 599.5



Rasterization

Shared Edges

- Each pixel needs to be rasterized exactly once
- Resulting image is independent of drawing order
- Rule: If pixel center exactly touches an edge or vertex
  - Fill pixel only if triangle extends to the right or down



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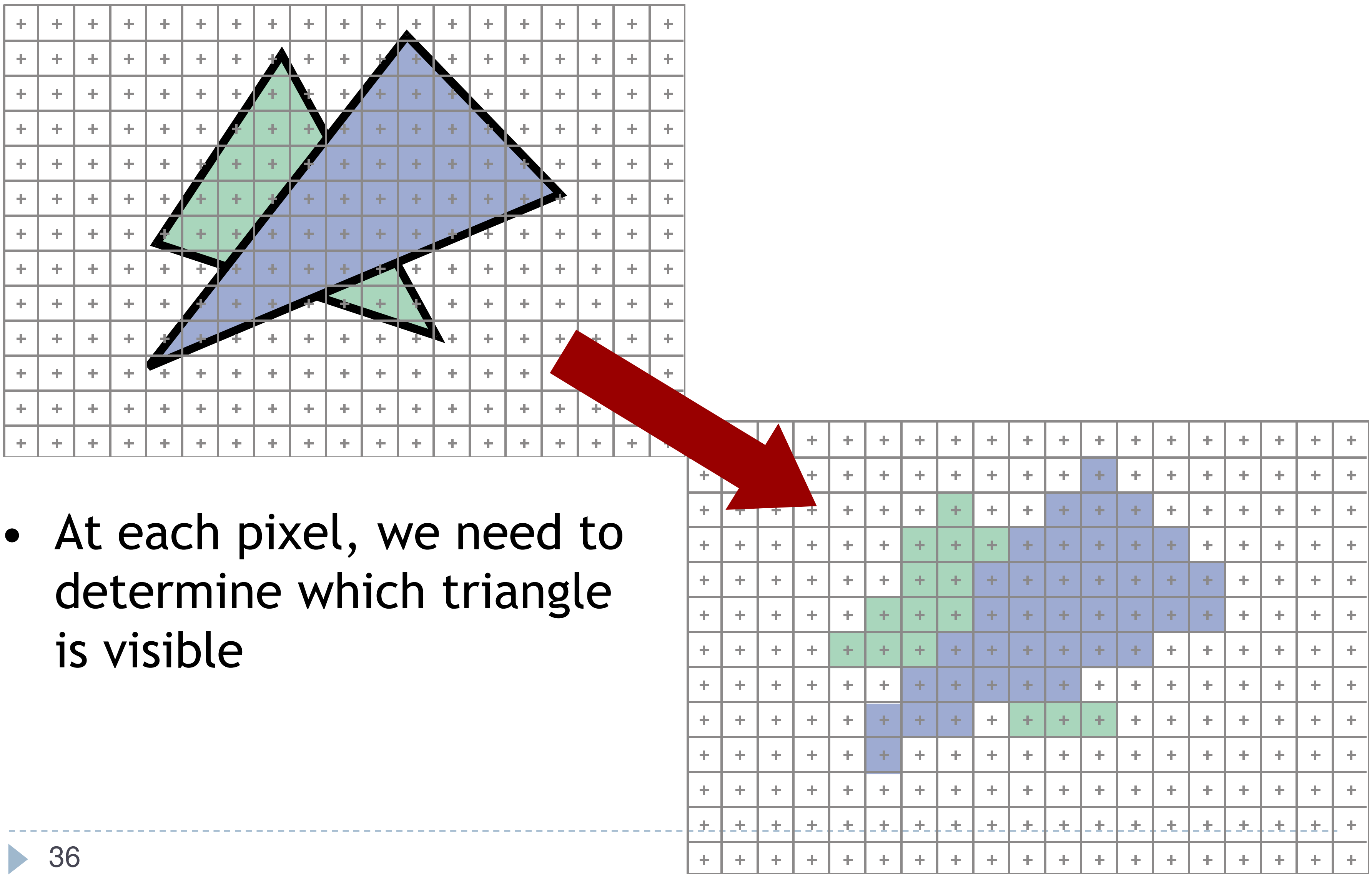
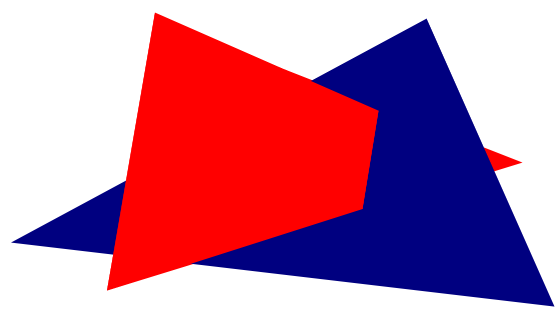
Visibility



- At each pixel, we need to determine which triangle is visible

Painter's Algorithm

- Paint from back to front
- Every new pixel always paints over previous pixel in frame buffer
- Need to sort geometry according to depth
- May need to split triangles if they intersect



- Old style, before memory became cheap

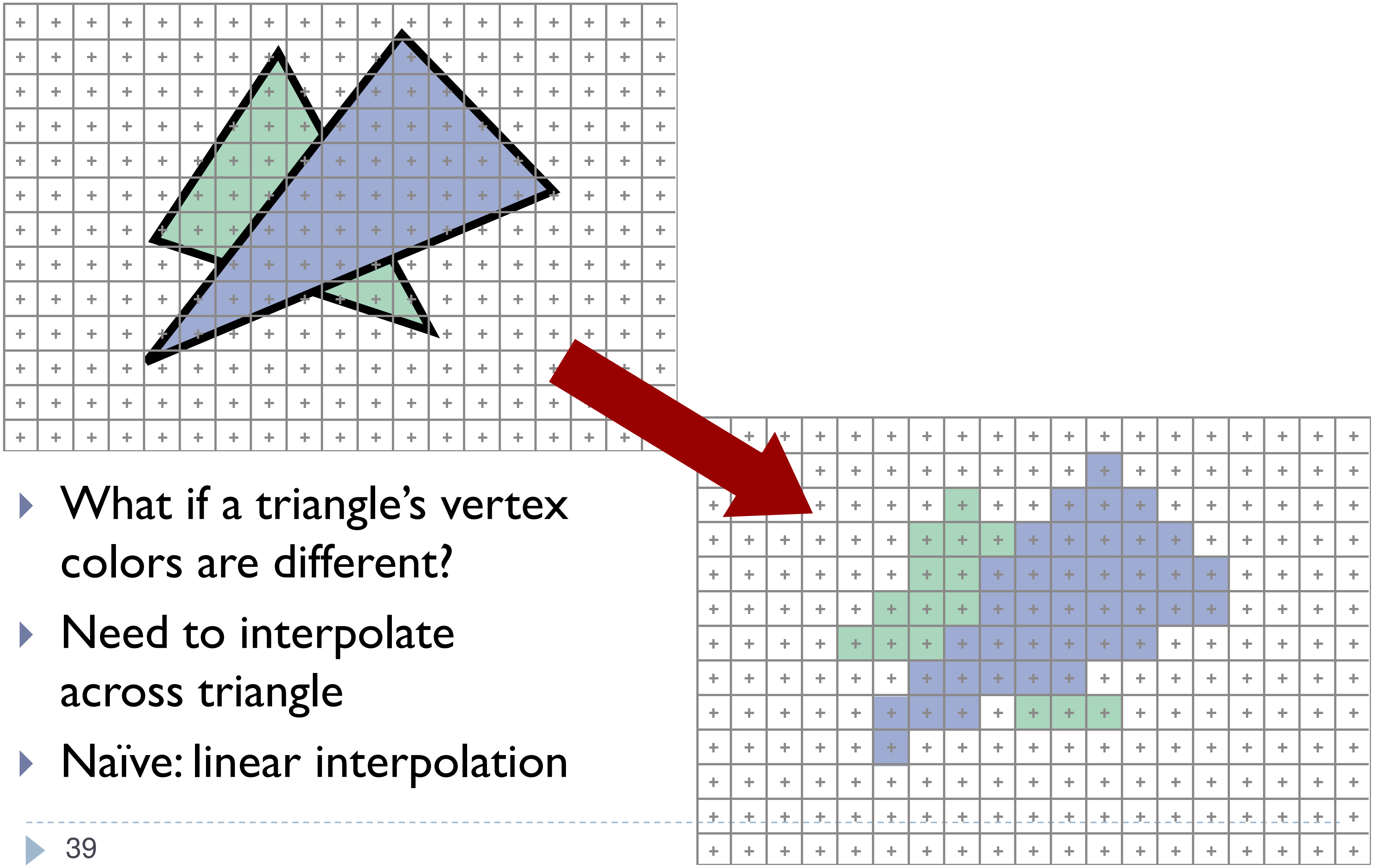
Z-Buffering

- Store z-value for each pixel
- Depth test
  - During rasterization, compare stored value to new value
  - Update pixel only if new value is smaller

```
setpixel(int x, int y, color c, float z)
if(z<zbuffer(x,y)) then
  zbuffer(x,y) = z
  color(x,y) = c
```

- z-buffer is dedicated memory reserved for GPU (graphics memory)
- Depth test is performed by GPU

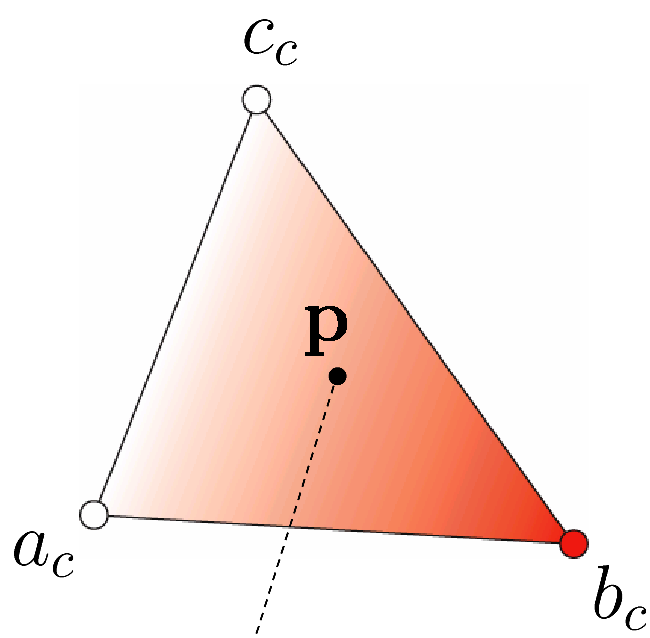
Rasterization



- What if a triangle's vertex colors are different?
- Need to interpolate across triangle
- Naïve: linear interpolation

Barycentric Interpolation

- Interpolate values across triangles, e.g., colors



$$c(\mathbf{p}) = \alpha(\mathbf{p})a_c + \beta(\mathbf{p})b_c + \gamma(\mathbf{p})c_c$$

- Linear interpolation on triangles
  - Barycentric coordinates

Next Lecture

- Perspectively Correct Interpolation
- Everything about color