CSE 167: Introduction to Computer Graphics Lecture #5: Color

Jürgen P. Schulze, Ph.D.
University of California, San Diego
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Announcements

- Homework project #2 due this Friday, October 8
  - To be presented between 1-4pm in lab 260.
  - This Friday:
    - Instructor will be present 1-3pm
    - TAs+tutors will be present 2-4pm
- Late submissions for project #1 accepted until Friday, October 8
- Homework #3 introduction Monday at 9:30am by Han
- Lab hour location changes with availability (Monday afternoon it will be 210)
- Need email address for Gradesource from:
  - Vitus Lorenz-Meyer

Lecture Overview

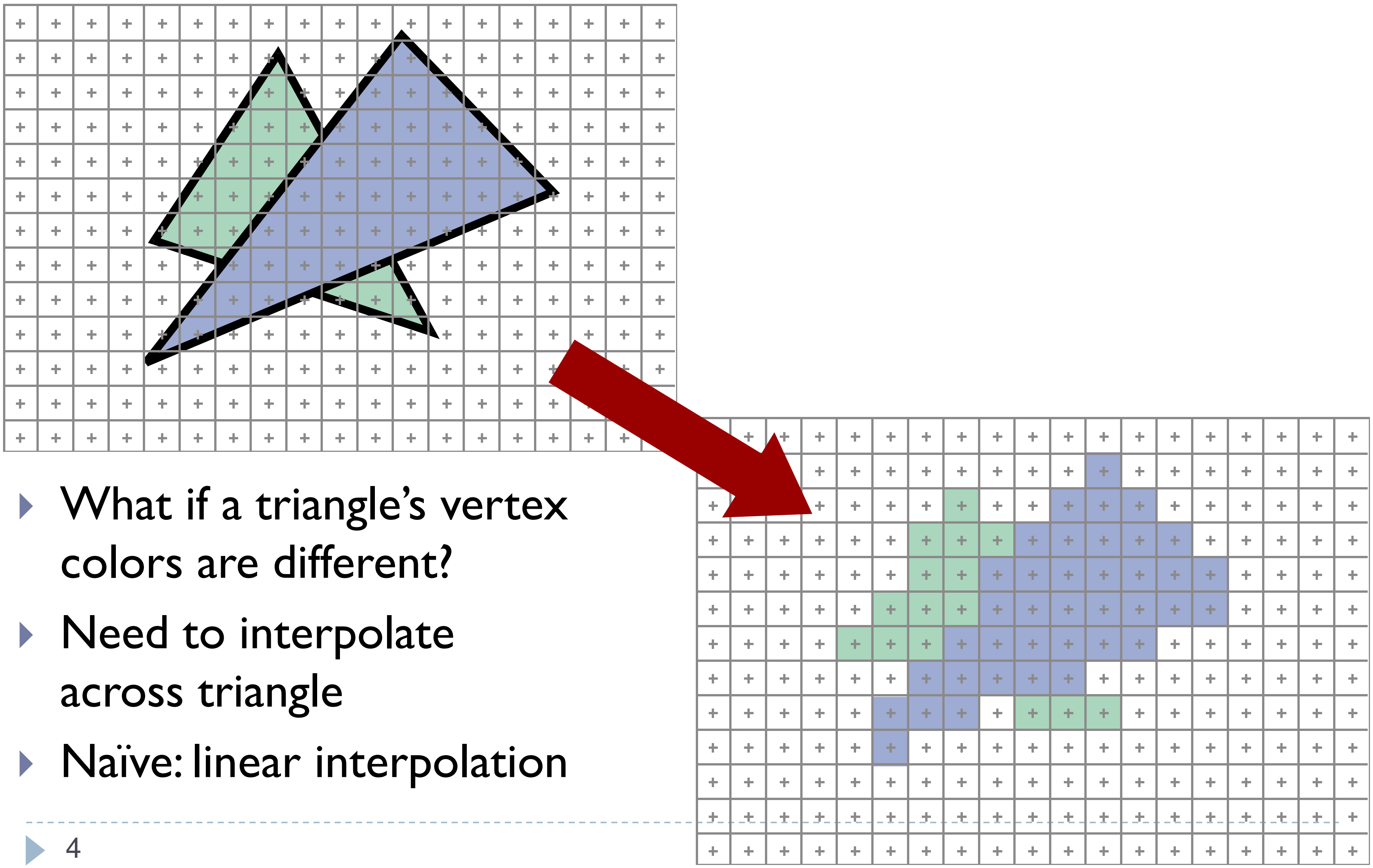
Rasterization

- Perspectively correct interpolation

Color

- Physical background
- Color perception
- Color spaces

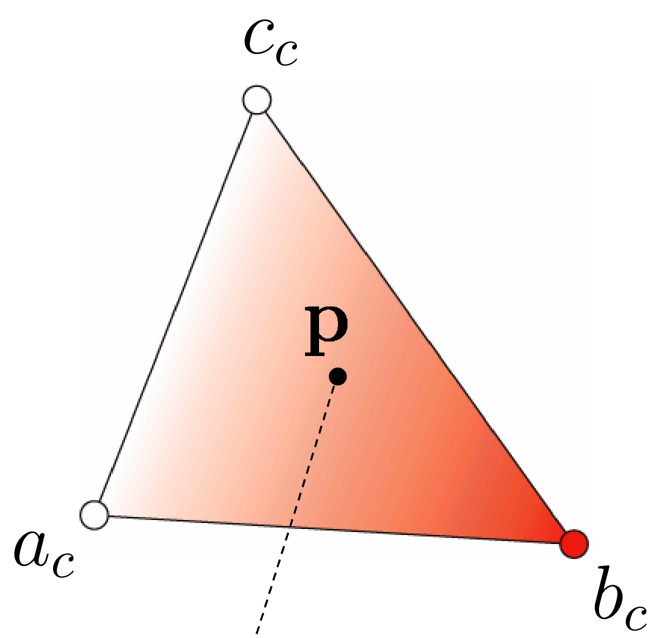
Rasterization



- What if a triangle's vertex colors are different?
- Need to interpolate across triangle
- Naïve: linear interpolation

Barycentric Interpolation

- Interpolate values across triangles, e.g., colors



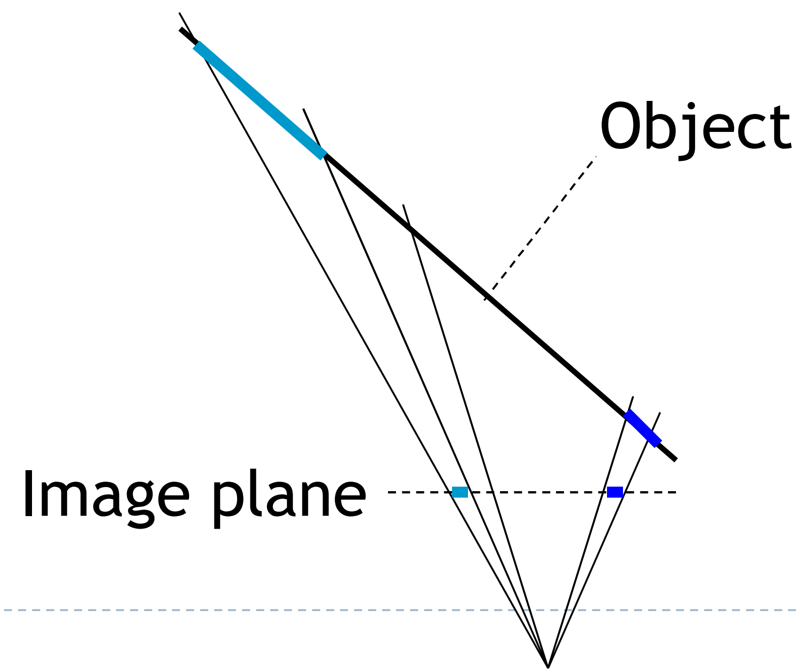
$$c(\mathbf{p}) = \alpha(\mathbf{p})a_c + \beta(\mathbf{p})b_c + \gamma(\mathbf{p})c_c$$

- Linear interpolation on triangles
  - Barycentric coordinates

Perspectively Correct Interpolation

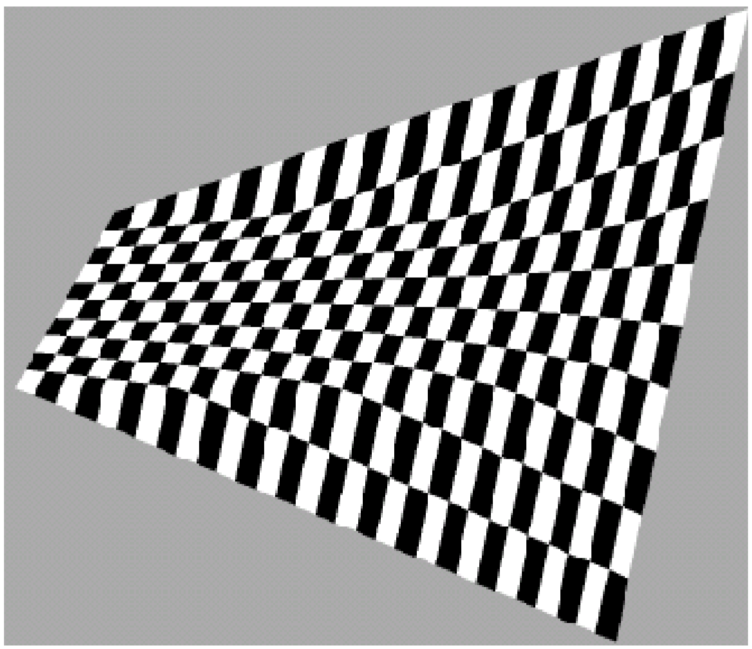
Problem

- Barycentric (linear) interpolation in image coordinates does not correspond to barycentric interpolation in camera space
- Equal step size on image plane does not correspond to equal step size on object

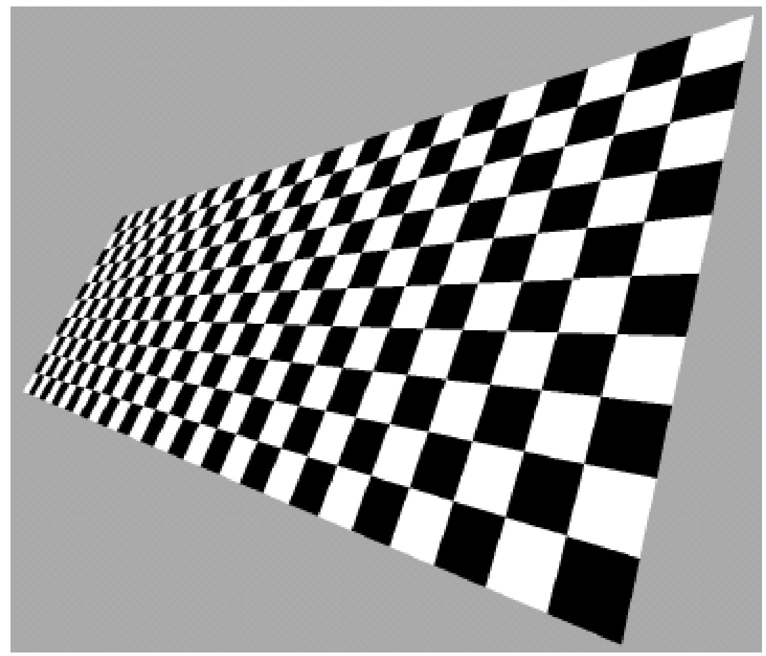


Perspectively Correct Interpolation

Linear interpolation
in image coordinates



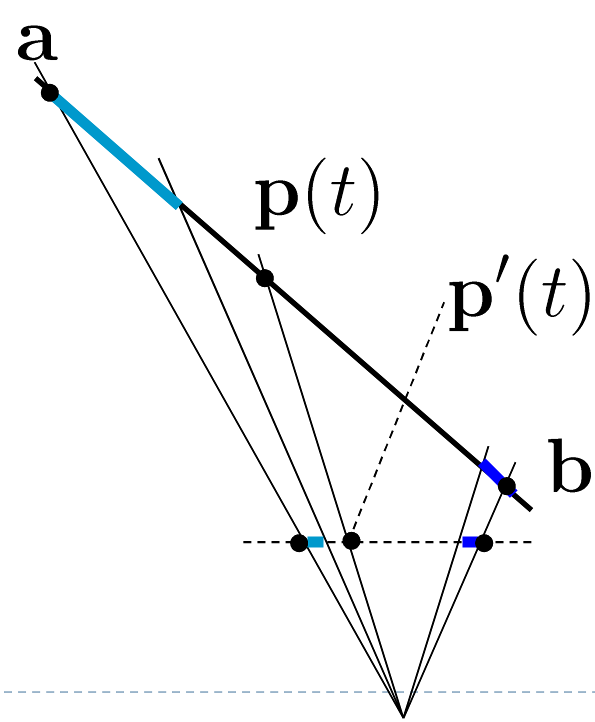
Perspectively correct
interpolation in
object coordinates



Perspective Projection Revisited

- Vertices **a**, **b** before projection
- Linear interpolation: $\mathbf{p}(t) = \mathbf{a} + t(\mathbf{b} - \mathbf{a})$
- Perspective projection, homogeneous division

$$\mathbf{p}'(t) = \frac{\mathbf{a} + t(\mathbf{b} - \mathbf{a})}{a_w + t(b_w - a_w)}$$



Perspective Projection Revisited

- Rewrite

$$\frac{\mathbf{a} + t(\mathbf{b} - \mathbf{a})}{a_w + t(b_w - a_w)} = \frac{\mathbf{a}}{a_w} + s(t)\left(\frac{\mathbf{b}}{b_w} - \frac{\mathbf{a}}{a_w}\right)$$

  with

$$s(t) = \frac{b_w t}{a_w + t(b_w - a_w)}$$

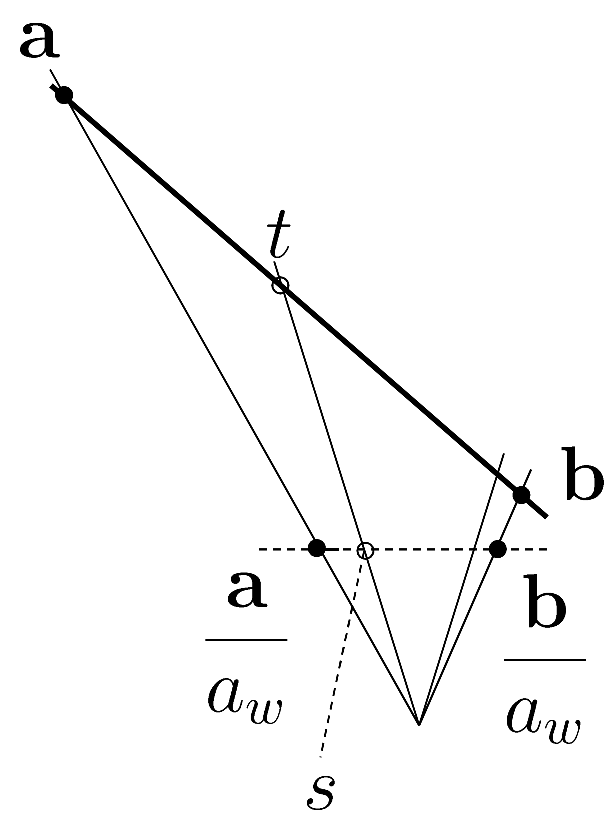
- *s* is linear interpolation weight in image space
- Straight lines are preserved
- Interpolation speed is different in *s* and *t*

Perspective Projection Revisited

- Relation between parameters

$$s(t) = \frac{b_w t}{a_w + t(b_w - a_w)}$$

$$t(s) = \frac{a_w s}{b_w + s(a_w - b_w)}$$



Perspective Projection Revisited

- Relation between parameters:

$$s(t) = \frac{b_w t}{a_w + t(b_w - a_w)}$$

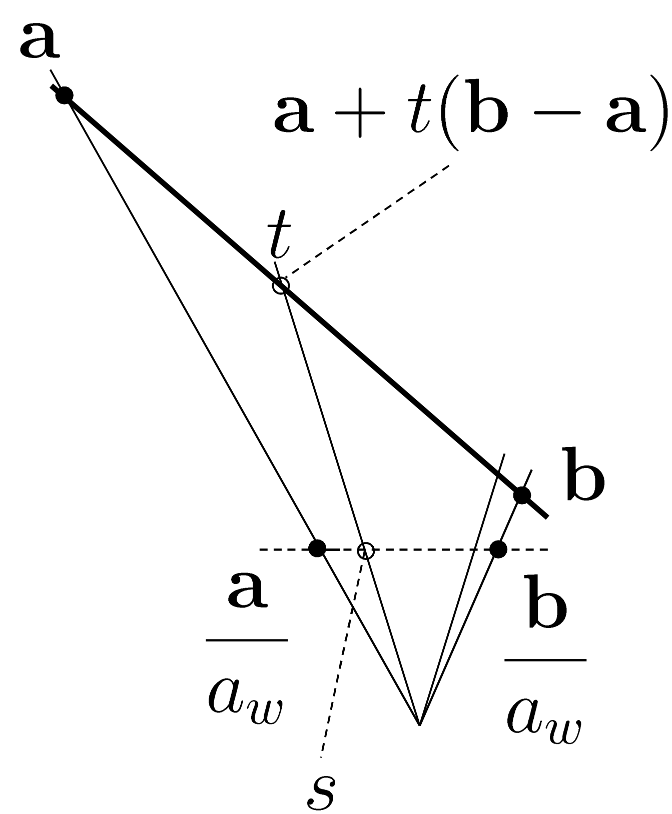
$$t(s) = \frac{a_w s}{b_w + s(a_w - b_w)}$$

- Projection after interpolation:

$$\frac{\mathbf{a} + t(\mathbf{b} - \mathbf{a})}{a_w + t(b_w - a_w)}$$

- Interpolation after projection:

$$\frac{\mathbf{a}}{a_w} + s\left(\frac{\mathbf{b}}{b_w} - \frac{\mathbf{a}}{a_w}\right)$$



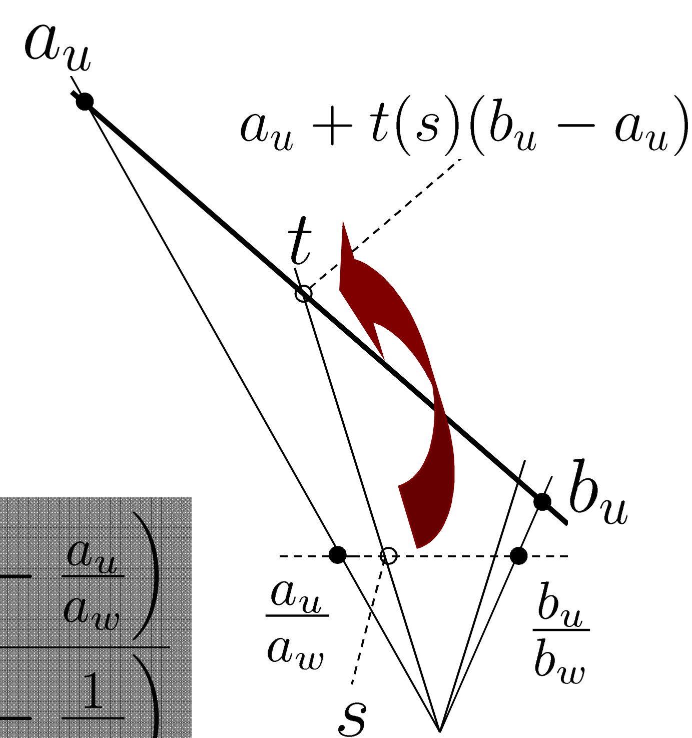
Perspectively Correct Interpolation

- In order to interpolate (in image space) any vertex attribute we need to compute a_u and b_u

- Hyperbolic interpolation:

$$a_u + t(s)(b_u - a_u)$$

$$a_u + t(s)(b_u - a_u) = \frac{\frac{a_u}{a_w} + s\left(\frac{b_u}{b_w} - \frac{a_u}{a_w}\right)}{\frac{1}{a_w} + s\left(\frac{1}{b_w} - \frac{1}{a_w}\right)}$$



Perspectively Correct Interpolation

Hyperbolic Interpolation

- Note $$\frac{1}{a_w} + s(t)\left(\frac{1}{b_w} - \frac{1}{a_w}\right) = \frac{1}{a_w + t(b_w - a_w)} \equiv \frac{1}{w}$$
- Recipe: given parameter s in image space

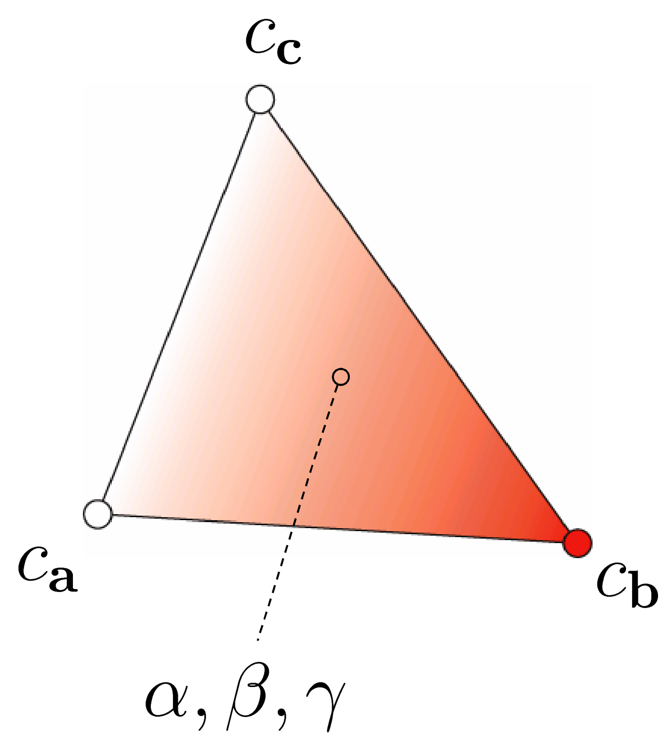
1.
2. $$\frac{1}{w} = \frac{1}{a_w} + s\left(\frac{1}{b_w} - \frac{1}{a_w}\right) = (1-s)\frac{1}{a_w} + s\frac{1}{b_w}$$
3. $$\frac{u}{w} = \frac{a_u}{a_w} + s\left(\frac{b_u}{b_w} - \frac{a_u}{a_w}\right) = (1-s)\frac{a_u}{a_w} + s\frac{b_u}{b_w}$$

$$u = \frac{u}{w} \Big/ \frac{1}{w}$$

Perspectively Correct Interpolation

- Works for triangles with barycentric coordinates
- Given point in image space with barycentric coordinates α, β, γ

1. $$\frac{1}{w} = \alpha \frac{1}{a_w} + \beta \frac{1}{b_w} + \gamma \frac{1}{c_w}$$
2. $$\frac{c}{w} = \alpha \frac{a_c}{a_w} + \beta \frac{b_c}{b_w} + \gamma \frac{c_c}{c_w}$$
3. $$c = \frac{c}{w} / \frac{1}{w}$$



Lecture Overview

Rasterization

- Perspectively correct interpolation

Color

- Physical background
- Color perception
- Color spaces

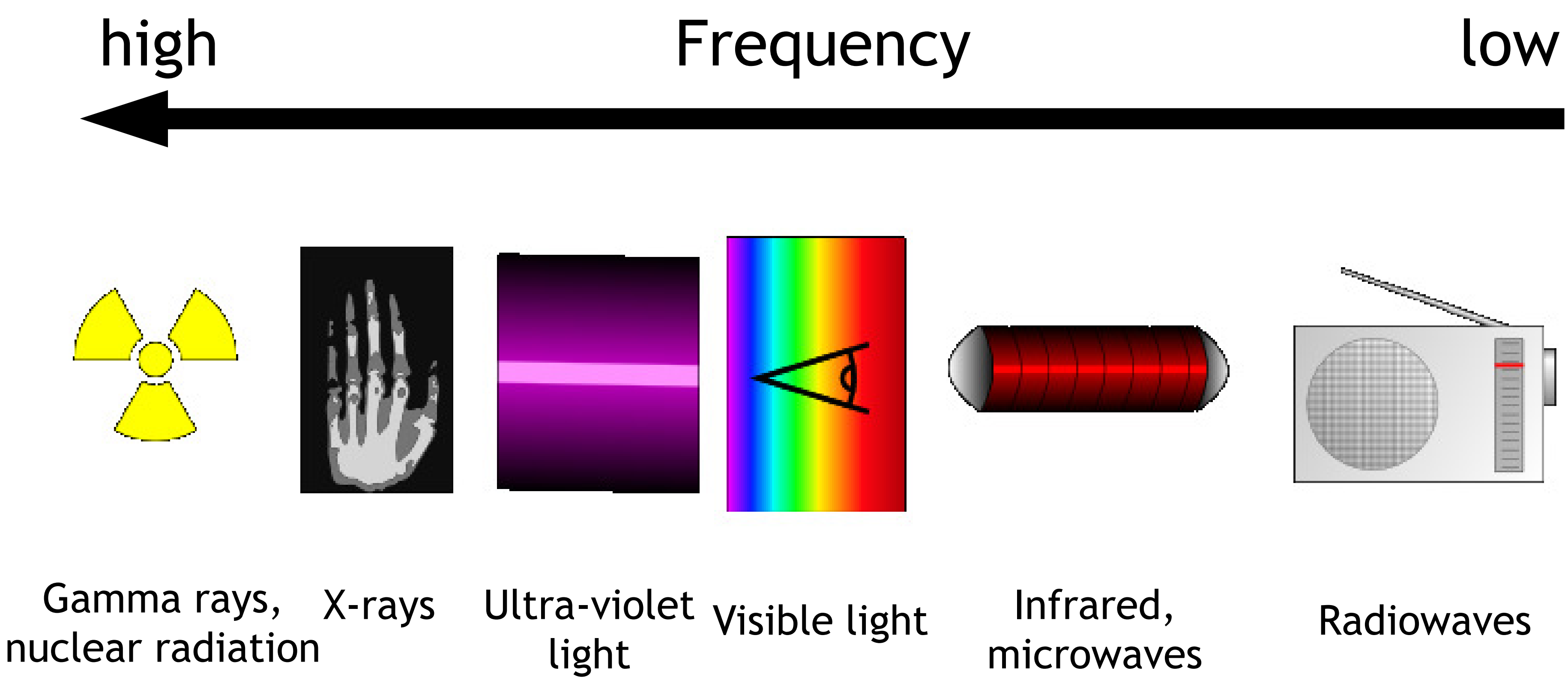
Light

Physical models

- Electromagnetic waves [Maxwell 1862]
- Photons (tiny particles) [Planck 1900]
- Wave-particle duality [Einstein, early 1900]
  "It depends on the experiment you are doing whether light behaves as particles or waves"
- Simplified models in computer graphics

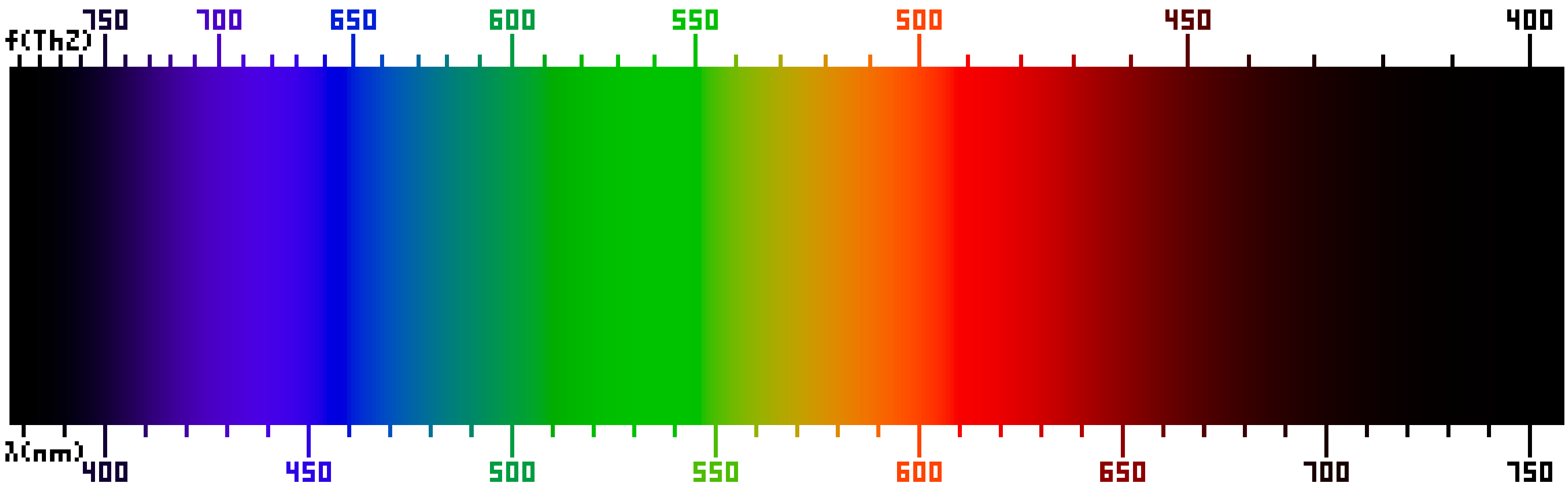
Electromagnetic Waves

- Different frequencies



Visible Light

Frequency



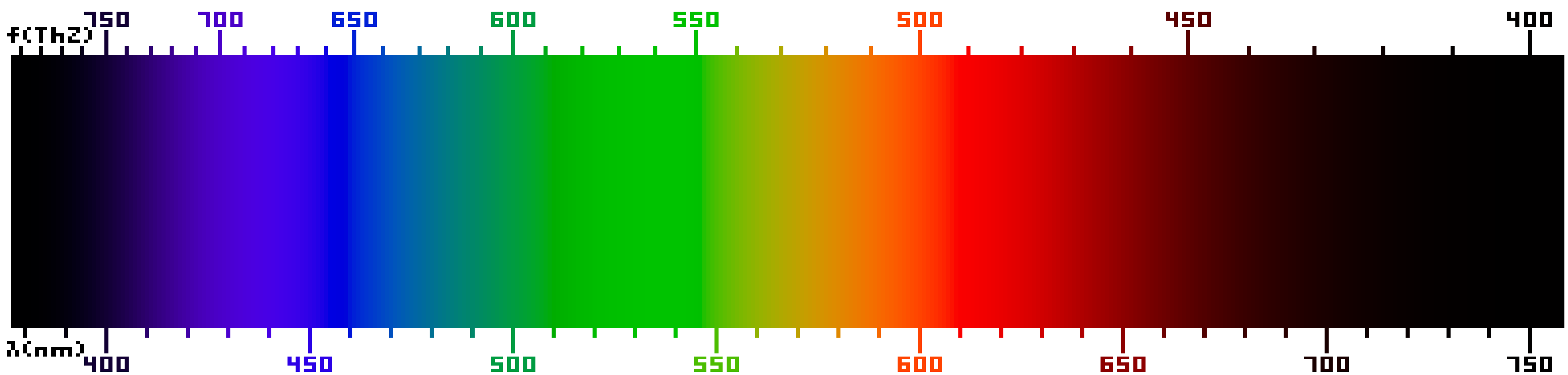
Wavelength: 1nm=10^-9 meters
speed of light = wavelength * frequency

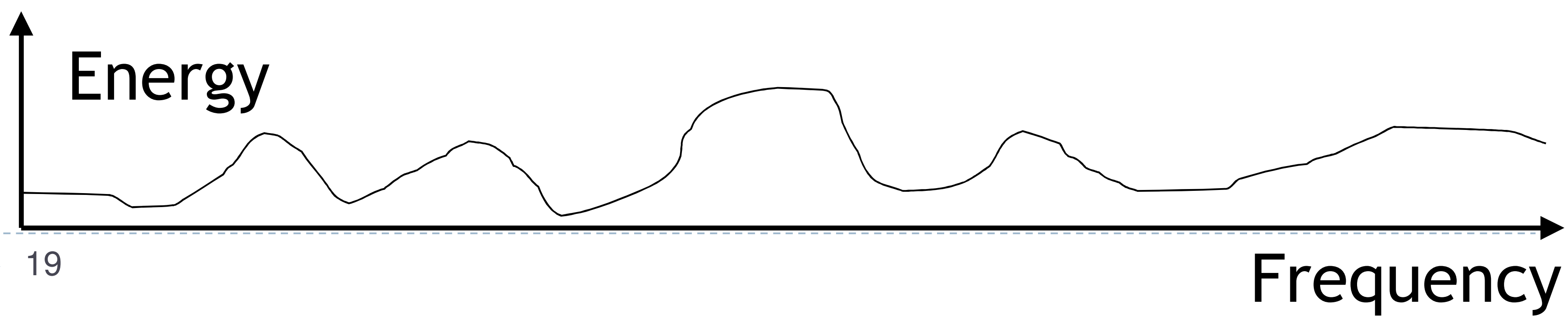
Example 94.9MHz: $$\frac{300*10^6\ \frac{m}{s}}{94.9*10^6\ \frac{1}{s}} = 3.16m$$

Light Transport

Simplified model in computer graphics

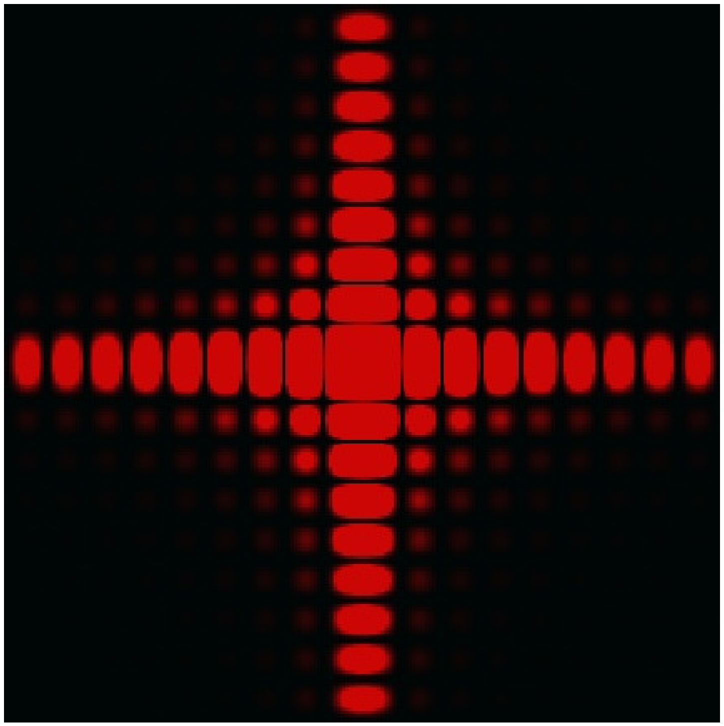
- Light is transported along straight rays
- Rays carry a spectrum of electromagnetic energy





Limitations

- Wave nature of light ignored
- E.g., no diffraction effects



Diffraction pattern of a small square aperture

Surface of a DVD forms a diffraction grating

Lecture Overview

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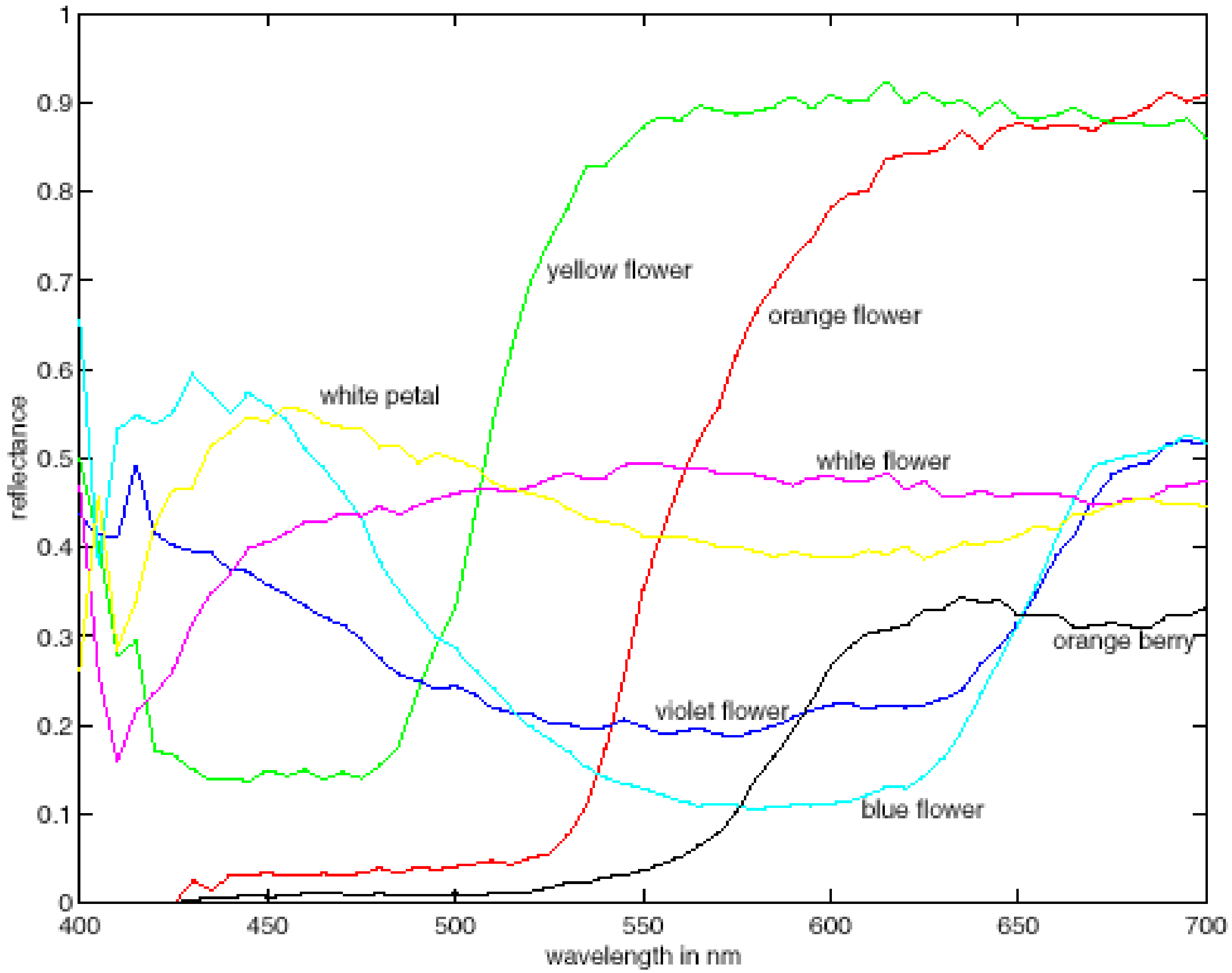
Light and Color

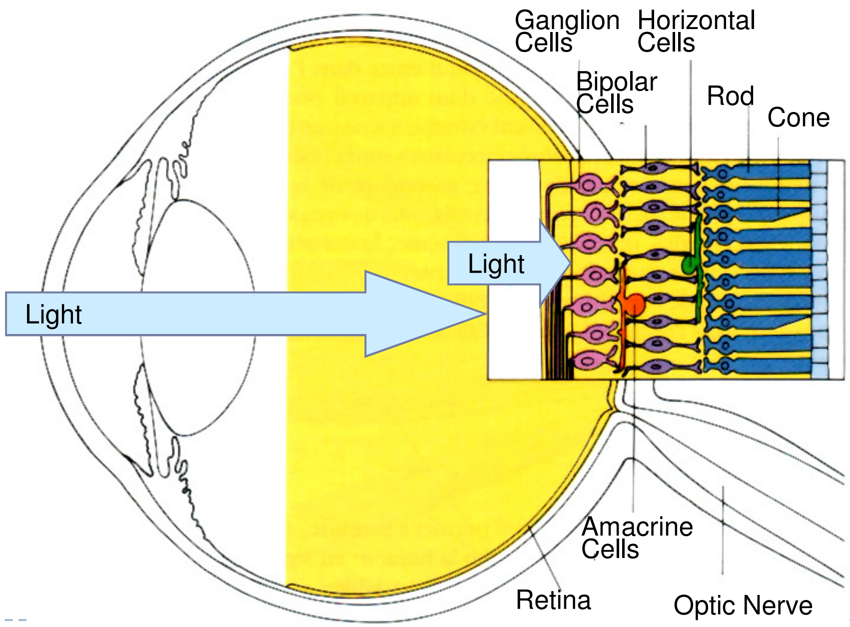
- Different spectra may be perceived as the same color

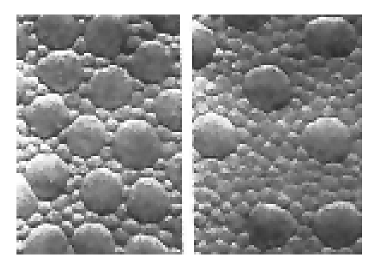


Color Perception

- Photoreceptor cells
- Light sensitive
- Two types, rods and cones


Rods Cones




Distribution of Cones and Rods

Photoreceptor Cells

Rods

- More than 1,000 times more sensitive than cones
- Low light vision
- Brightness perception only, no color
- Predominate in peripheral vision

Cones

- Responsible for high-resolution vision
- 3 types of cones for different wavelengths (LMS):
  - L: long, red
  - M: medium, green
  - S: short, blue

Photoreceptor Cells

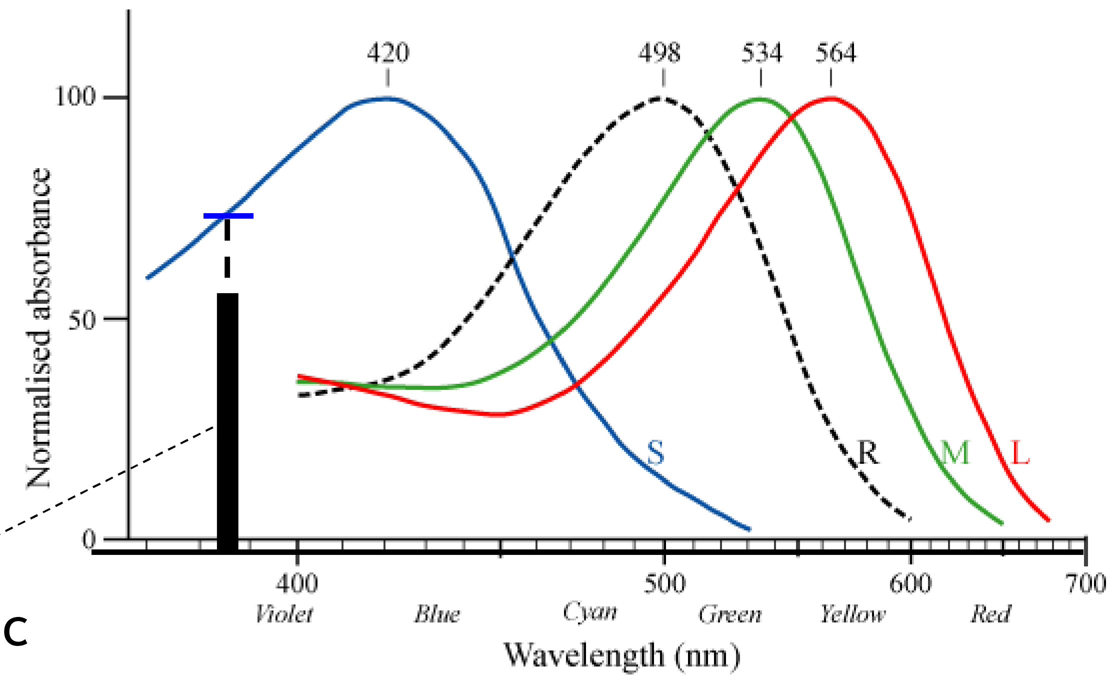
(Source: Encyclopedia Britannica)

The Austrian naturalist Karl von Frisch has demonstrated that honeybees, although blind to red light, distinguish at least four different color regions, namely:

- yellow (including orange and yellow green)
- blue green
- blue (including purple and violet)
- ultraviolet

Photoreceptor Cells

- Response curves $s(\lambda), m(\lambda), l(\lambda)$ to monochromatic spectral stimuli



Monochromatic stimulus

- Experimentally determined in the 1980s

Photoreceptor Cells

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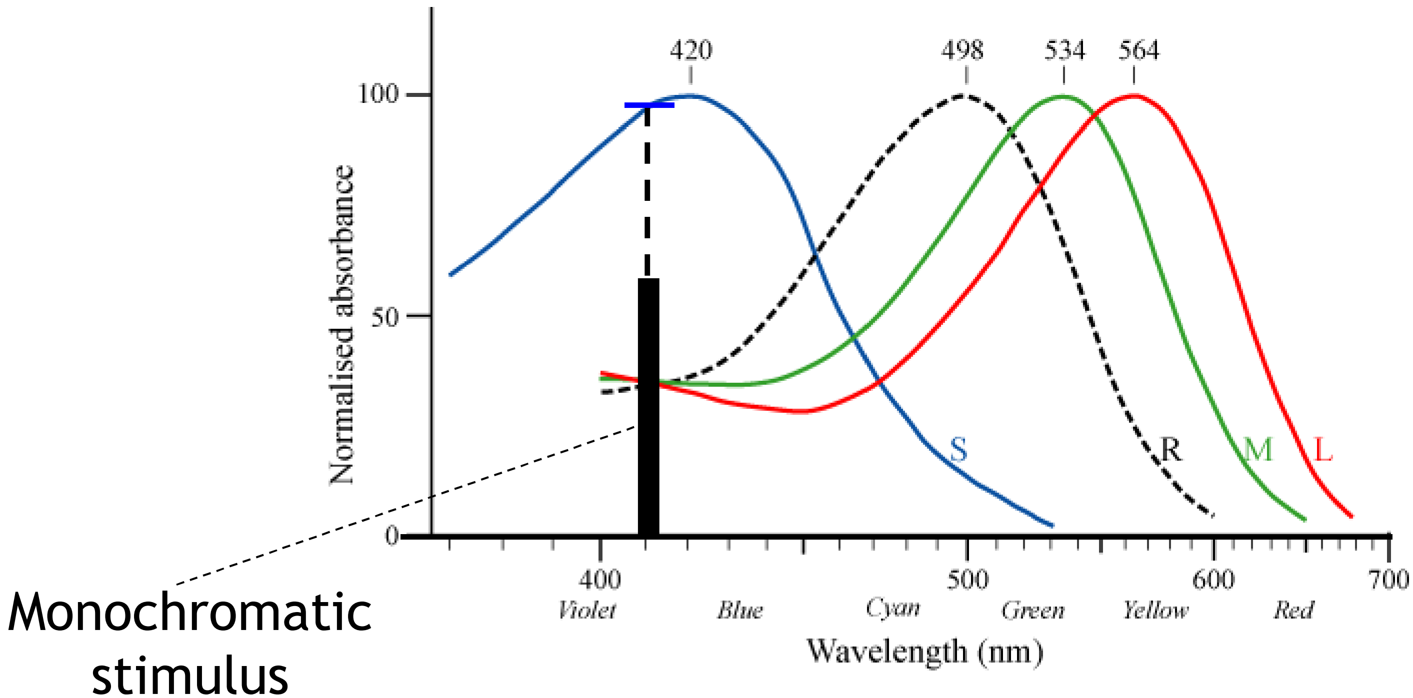

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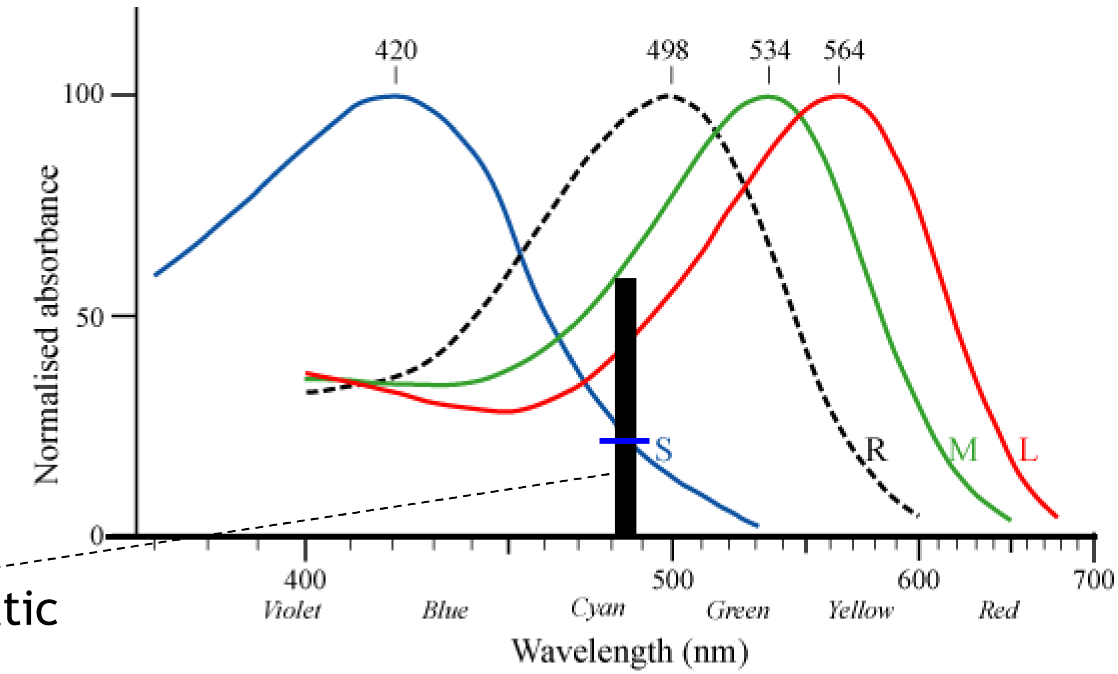
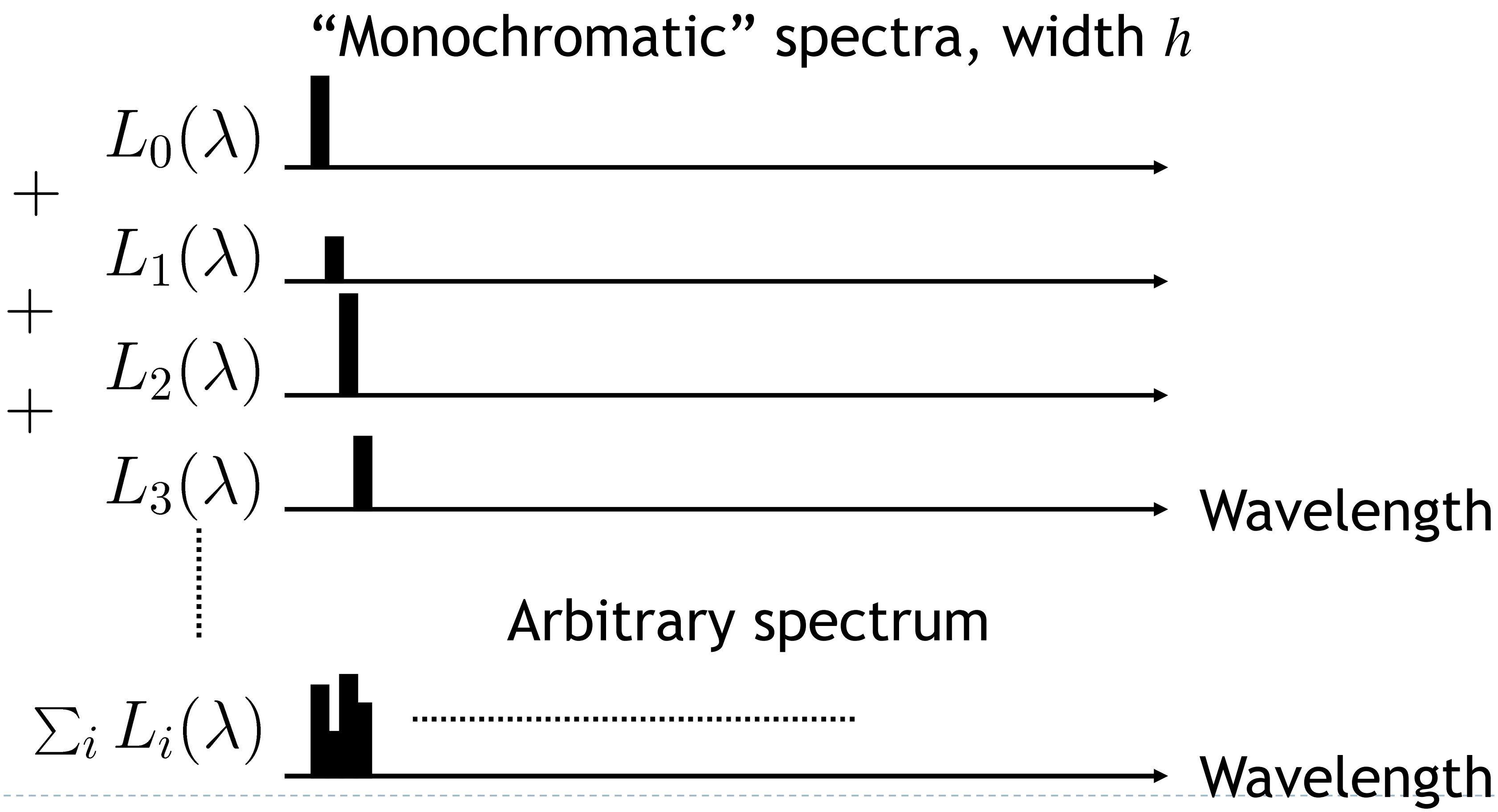


Monochromatic stimulus

- Experimentally determined in the 1980s

Response to Arbitrary Spectrum

- Arbitrary spectrum as sum of "mono-chromatic" spectra



Response to Arbitrary Spectrum

Assume linearity (superposition principle)

- Response to sum of spectra is equal to sum of responses to each spectrum
- S-cone $\text{response}_s = \sum_i s(\lambda) h L_i(\lambda)$

  Input: light intensity $L(\lambda)$ impulse width *h*
  Response to monochromatic impulse $s(\lambda)$
- In the limit $h \rightarrow 0$

$$\text{response}_s = \int s(\lambda) L(\lambda) d\lambda$$

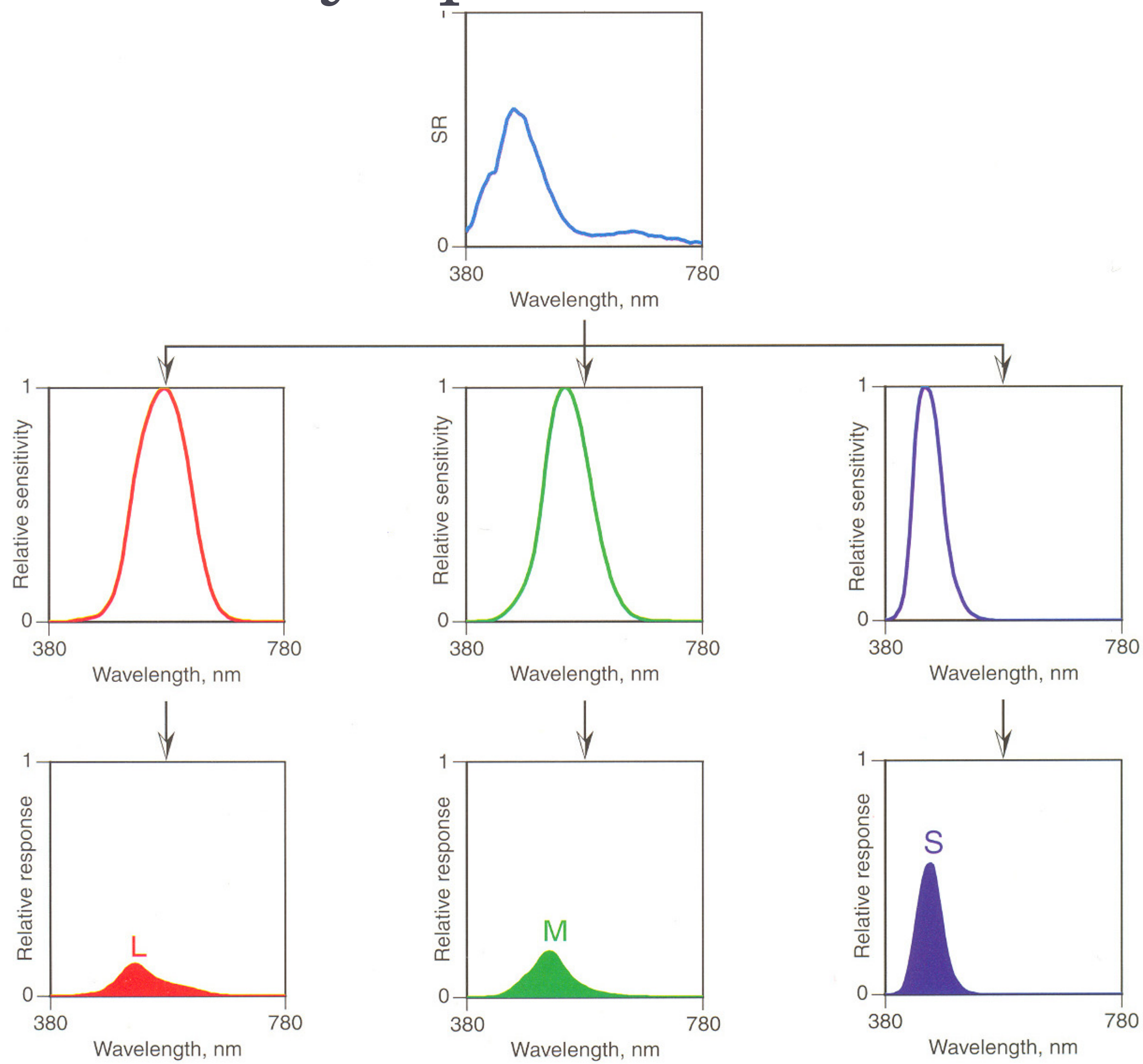
Response to Arbitrary Spectrum

Stimulus

Response curves

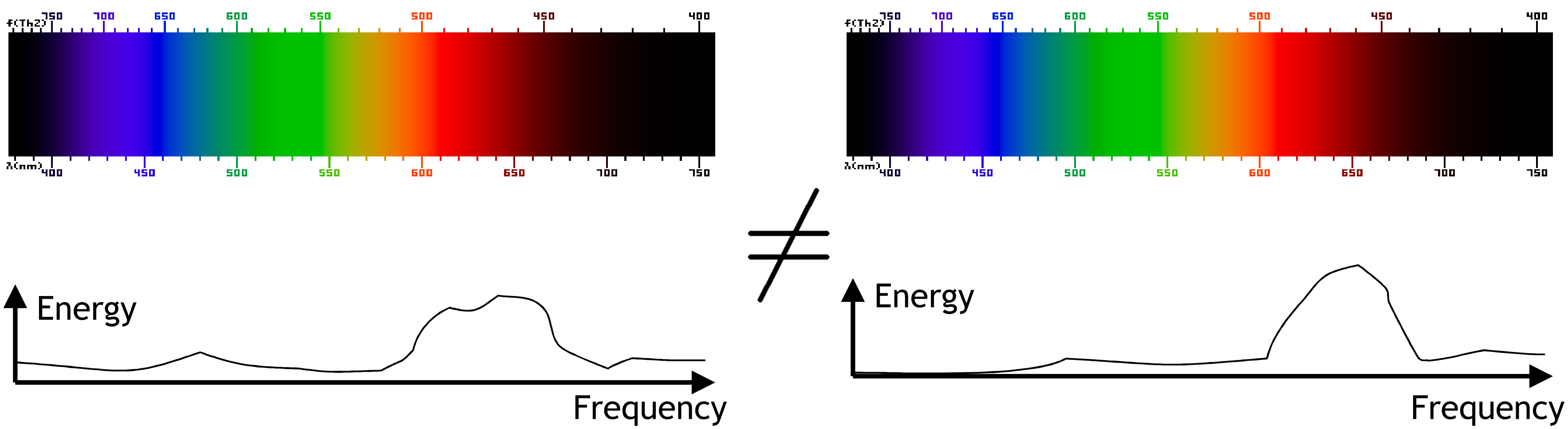
Multiply

Integrate



Metamers

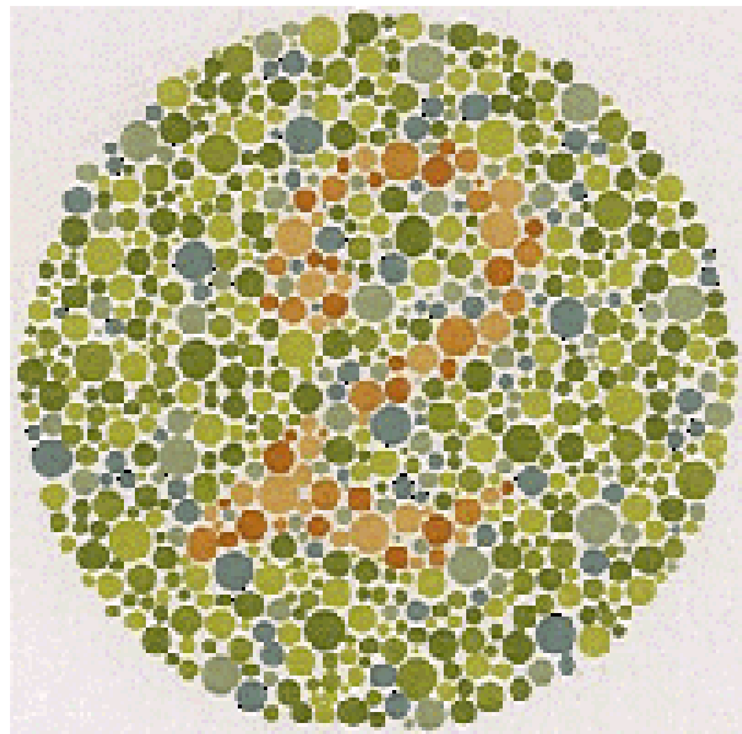
- Different spectra, same response
- Cannot distinguish spectra
  - Arbitrary spectrum is *infinitely dimensional* (has infinite number of degrees of freedom)
  - Response has three dimensions
  - Information is lost



Perceived color: red = Perceived color: red

Color Blindness

- One type of cone missing, damaged
- Different types of color blindness, depending on type of cone
- Can distinguish even fewer colors
- But we are all a little color blind…



Lecture Overview

Rasterization

- Perspectively correct interpolation

Color

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- Color perception
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Color Reproduction

- How can we reproduce, represent color?
  - One option: store full spectrum
- Representation should be as compact as possible
- Any pair of colors that can be distinguished by humans should have two different representations

Color Spaces

- Set of parameters describing a color sensation
- “Coordinate system” for colors
- Three types of cones, expect three parameters to be sufficient

Color Spaces

- Set of parameters describing a color sensation
- “Coordinate system” for colors
- Three types of cones, expect three parameters to be sufficient
- Why not use L,M,S cone responses?
  - Not known until 1980s

Trichromatic Theory

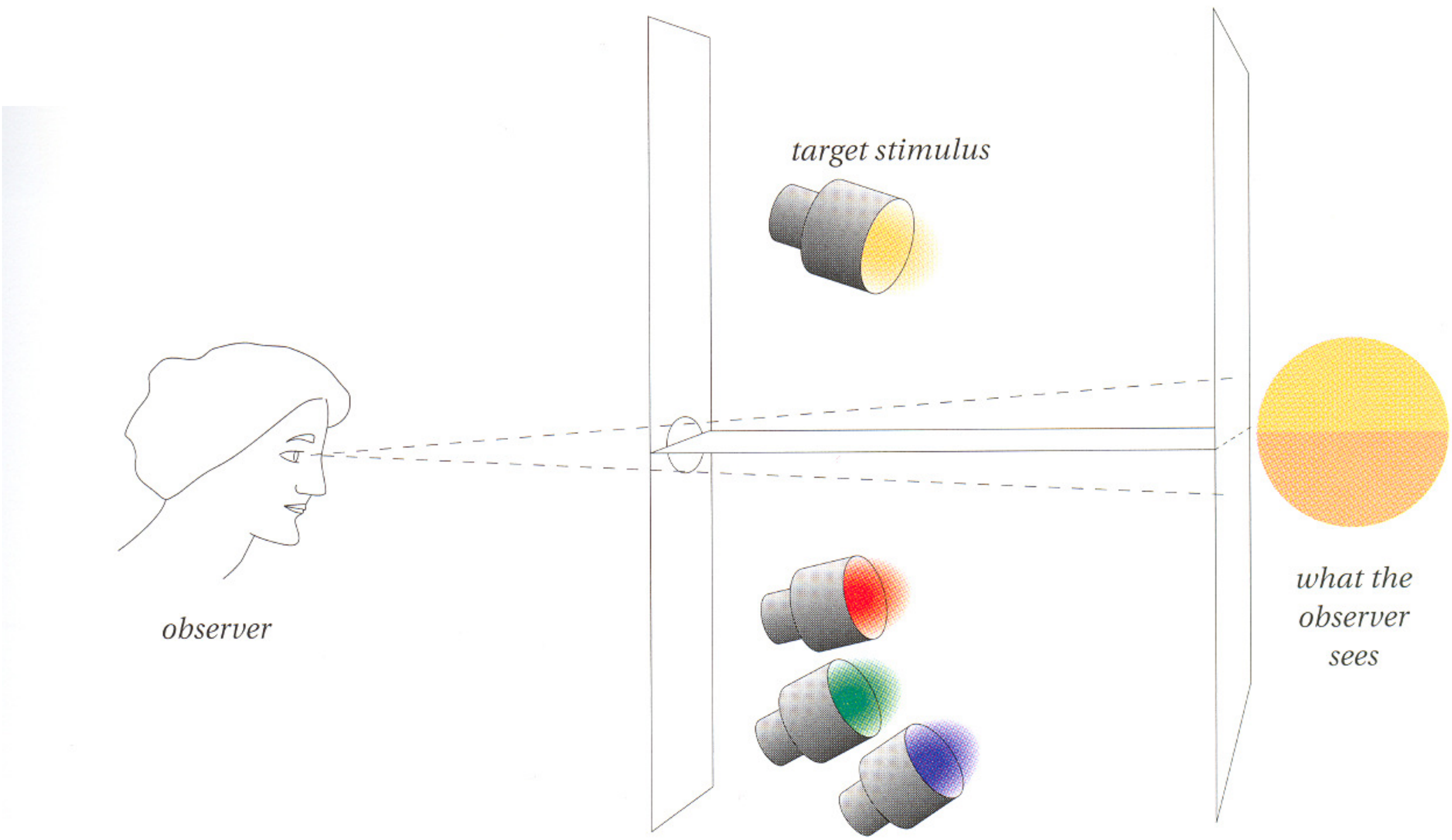
- Claims any color can be represented as a weighted sum of three primary colors
- Propose red, green, blue as primaries
- Developed in 18^{th}, 19^{th} century, before discovery of photoreceptor cells (Thomas Young, Hermann von Helmholtz)

Tristimulus Experiment

- Given arbitrary color, want to know the weights for the three primaries
- Tristimulus value
- Find out experimentally
  - CIE (Commission Internationale de l'Eclairage, International Commission on Illumination), circa 1920

Tristimulus Experiment

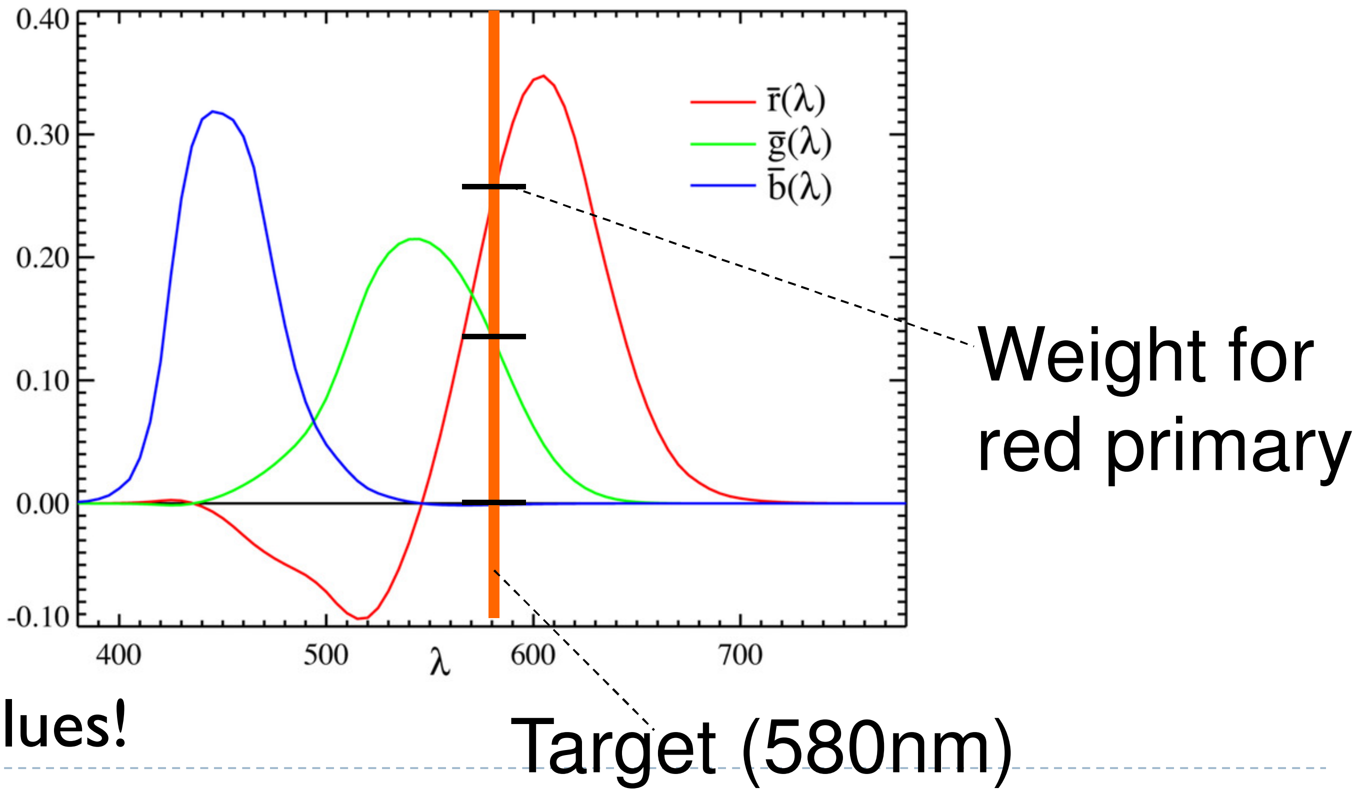
- Determine tristimulus values for spectral colors experimentally



The observer adjusts the intensities of the red, green, and blue lamps until they match the target stimulus on the split screen.

Tristimulus Experiment

- Spectral primary colors were chosen
  - Blue (435.8nm), green (546.1nm), red (700nm)
- Matching curves for monochromatic target



- Negative values!

Tristimulus Experiment

Negative values

- Some spectral colors could not be matched by primaries in the experiment
- "Trick"
  - One primary could be added to the source (stimulus)
  - Match with the other two
  - Weight of primary added to the source is considered negative

Photoreceptor response vs. matching curve

- **Not the same!**

Tristimulus Values

- Given arbitrary spectrum, find weights of primaries such that weighted sum of primaries is perceived the same as input spectrum
- Linearity again
  - Matching values for a sum of spectra with small spikes are the same as sum of matching values for the spikes
  - In the limit (spikes are infinitely narrow)

$$R = \int \bar{r}(\lambda) L(\lambda) d\lambda$$

$$G = \int \bar{g}(\lambda) L(\lambda) d\lambda$$

$$B = \int \bar{b}(\lambda) L(\lambda) d\lambda$$

  - Monochromatic matching curves $\bar{r}(\lambda), \bar{g}(\lambda), \bar{b}(\lambda)$

CIE Color Spaces

- Matching curves $\bar{r}(\lambda), \bar{g}(\lambda), \bar{b}(\lambda)$ define CIE RGB color space
  - CIE RGB values are color "coordinates"
- CIE was not satisfied with range of RGB values for visible colors
- Defined CIE XYZ color space
- Most commonly used color space today

CIE XYZ Color Space

- Linear transformation of CIE RGB

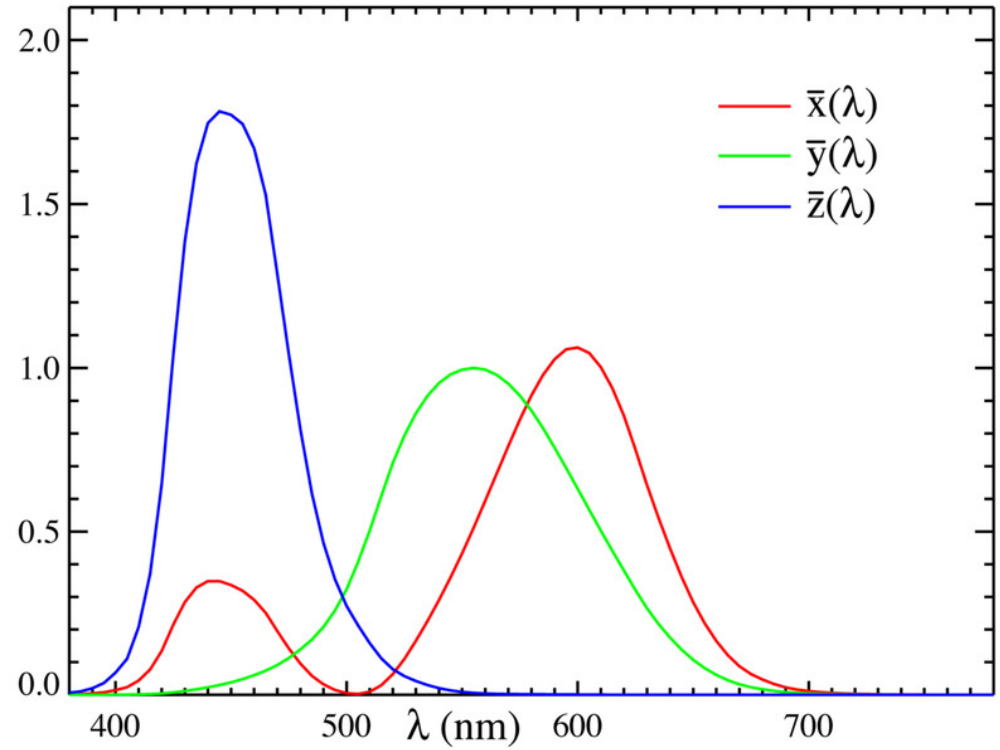
$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \frac{1}{b_{21}} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix} = \frac{1}{0.17697} \begin{bmatrix} 0.49 & 0.31 & 0.20 \\ 0.17697 & 0.81240 & 0.01063 \\ 0.00 & 0.01 & 0.99 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

- Determined coefficients such that
  - Y corresponds to an experimentally determined brightness
  - No negative values in matching curves
  - White is XYZ=(1/3,1/3,1/3)

CIE XYZ Color Space

Matching curves

- No corresponding physical primaries



Tristimulus values

- Always positive!

$$X = \int \bar{x}(\lambda) L(\lambda) d\lambda$$

$$Y = \int \bar{y}(\lambda) L(\lambda) d\lambda$$

$$Z = \int \bar{z}(\lambda) L(\lambda) d\lambda$$

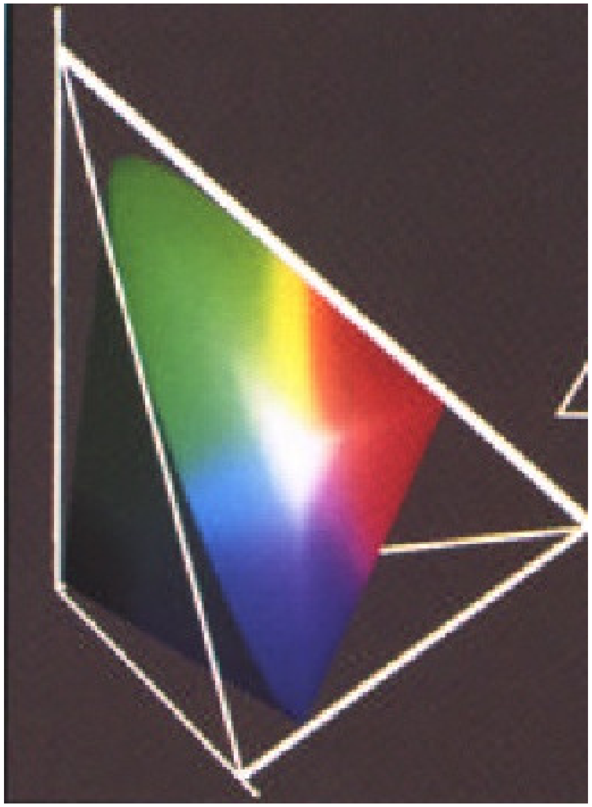
Summary

- CIE color spaces are defined by matching curves
  - At each wavelength, matching curves give weights of primaries needed to produce color perception of that wavelength
  - CIE RGB matching curves determined using trisimulus experiment
- Each distinct color perception has unique coordinates
  - CIE RGB values may be negative
  - CIE XYZ values are always positive

CIE XYZ Color Space

Visualization

- Interpret XYZ as 3D coordinates
- Plot corresponding color at each point
- Many XYZ values do not correspond to visible colors

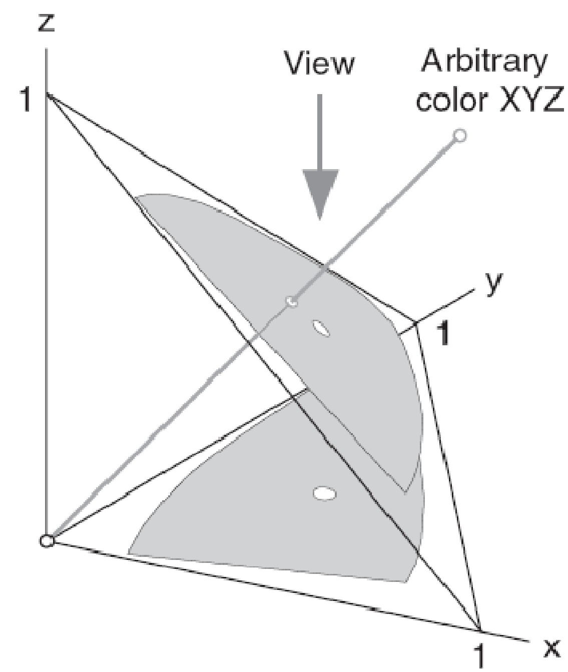


Chromaticity Diagram

- Project from XYZ coordinates to 2D for more convenient visualization

$$x = \frac{X}{X+Y+Z} \quad y = \frac{Y}{X+Y+Z} \quad z = \frac{Z}{X+Y+Z}$$

- Drop z-coordinate



Chromaticity Diagram

- Factor out luminance (perceived brightness) and chromaticity (hue)
  - x,y represent chromaticity of a color

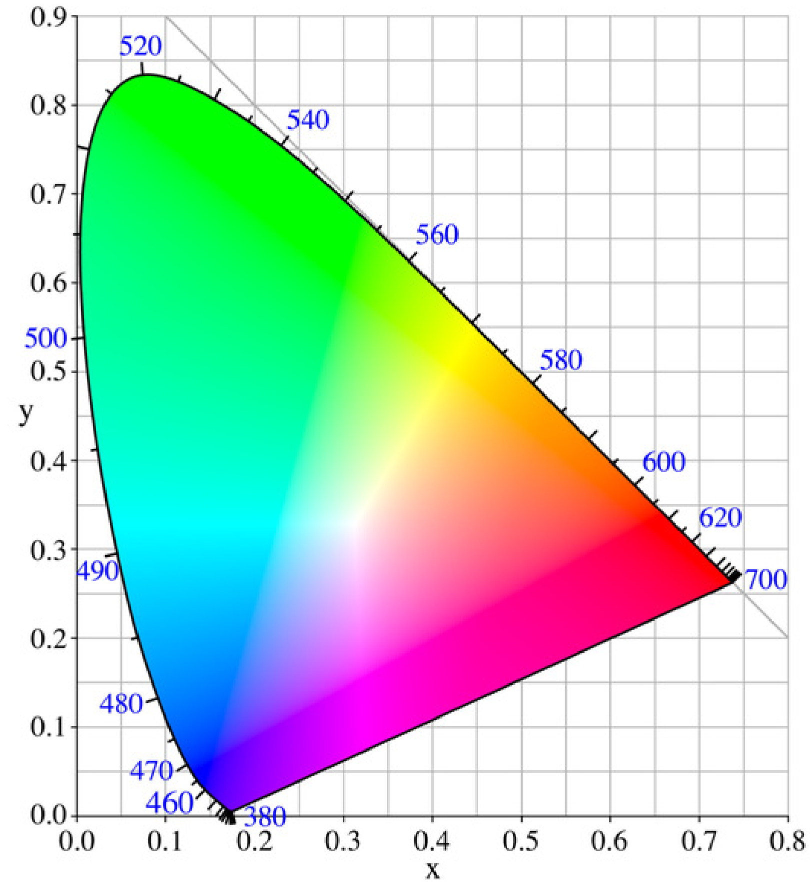
$$x = \frac{X}{X+Y+Z} \quad y = \frac{Y}{X+Y+Z} \quad 0 \leq x, y \leq 1$$

  - Y is luminance
- CIE xyY color space
- Reconstruct XYZ values from xyY

$$X = \frac{Y}{y}x \qquad Z = \frac{Y}{y}(1 - x - y)$$

Chromaticity Diagram

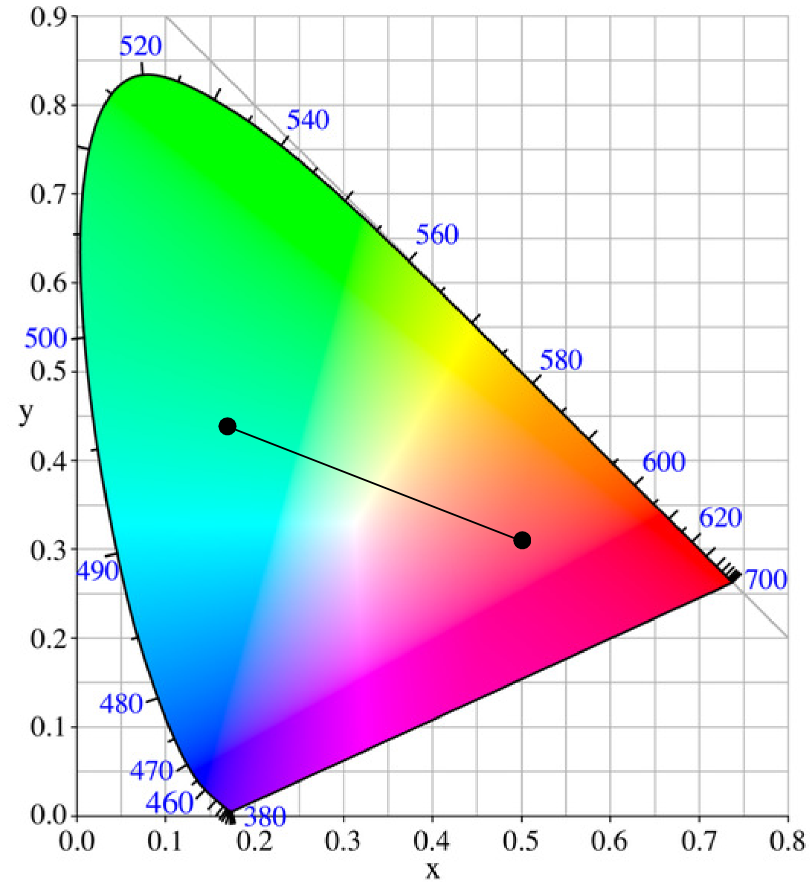
- Visualizes x,y plane (chromaticities)
- Pure spectral colors on boundary



Colors shown do not correspond to colors represented by (x,y) coordinates!

Chromaticity Diagram

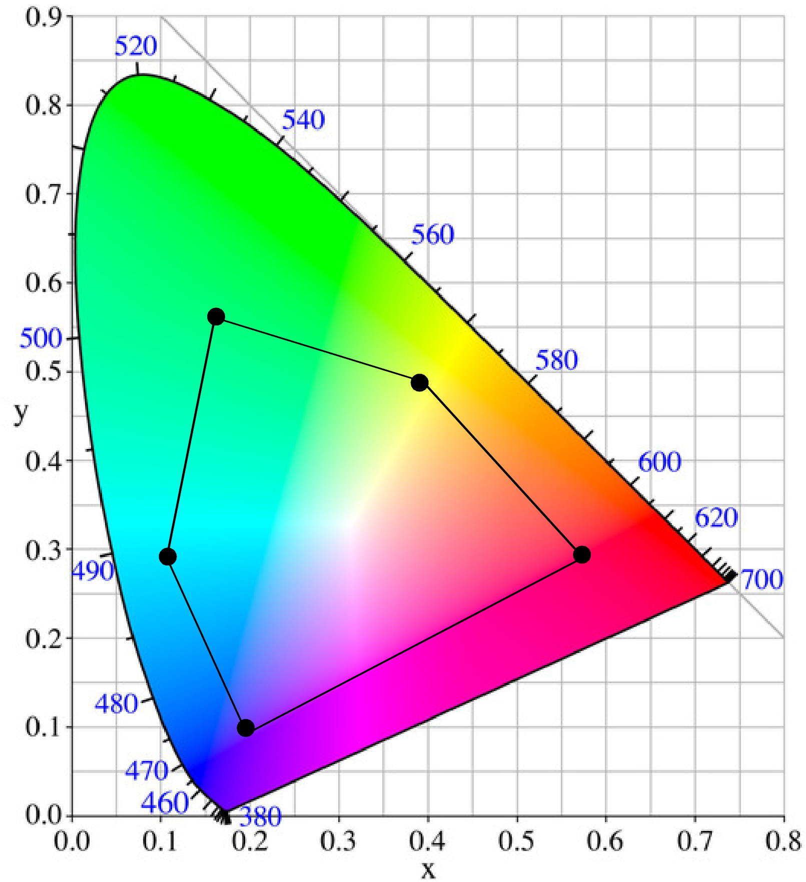
- Visualizes x,y plane (chromaticities)
- Pure spectral colors on boundary
- Weighted sum of any two colors lies on line connecting colors



Colors shown do not correspond to colors represented by (x,y) coordinates!

Chromaticity Diagram

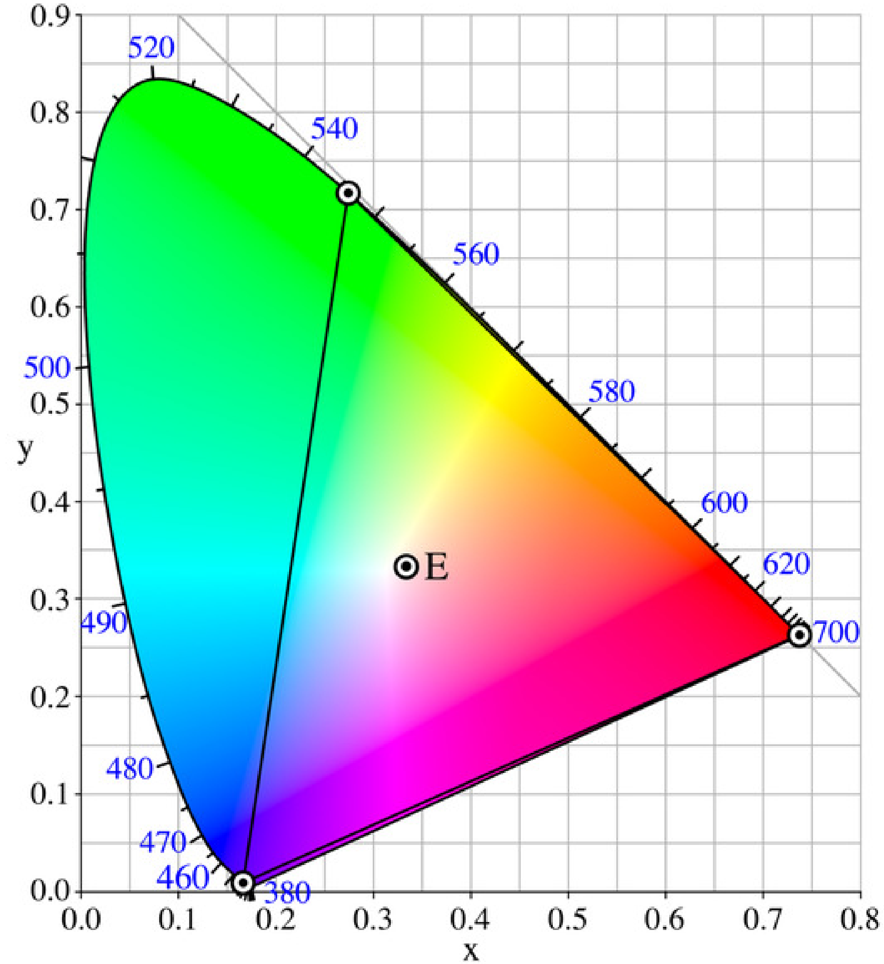
- Visualizes x,y plane (chromaticities)
- Pure spectral colors on boundary
- Weighted sum of any two colors lies on line connecting colors
- Weighted sum of any number of colors lies in convex hull of colors (gamut)



Colors shown do not correspond to colors represented by (x,y) coordinates!

Gamut

- Any device based on three primaries can only produce colors within the triangle spanned by the primaries
- Points outside gamut correspond to negative weights of primaries



Gamut of CIE RGB primaries

Next Lecture

- Shading