CSE167, Introduction to Computer Graphics
Midterm Exam, Tuesday May 6, 2008

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Please include all steps of your derivations in your answers to show your understanding of the problem. Try not to write more than the recommended amount of text. If your answer is a mix of correct and substantially wrong arguments we will consider deducting points for incorrect statements. There are twelve questions for a total score of 100 points.

Your name:

1. Compute the dot-product of the two vectors $\mathbf{a} = (2, 1, 6)$ and $\mathbf{b} = (5, 2, -2)$. What is the angle between $\mathbf{a}$ and $\mathbf{b}$? **6 points**

2. Given two vectors $\mathbf{a} = (3, 1, 5)$ and $\mathbf{b} = (2, 0, 4)$. Compute the cross-products $\mathbf{a} \times \mathbf{b}$ and $\mathbf{b} \times \mathbf{a}$. Compute the area of the parallelogram spanned by $\mathbf{a}$ and $\mathbf{b}$. **6 points**

3. How many basis vectors do you need to define a coordinate system for a two-dimensional plane, and how many for three-dimensional space? What is the problem if you use not enough basis vectors? What is the problem if you use too many? Do the basis vectors need to be perpendicular to each other? Explain why. (4-6 sentences) **8 points**

4. Given a translation and a scaling matrix $\mathbf{T}$ and $\mathbf{S}$,

$$
\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & t_{14} \\ 0 & 1 & 0 & t_{24} \\ 0 & 0 & 1 & t_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix}, , \quad
\mathbf{S} = \begin{bmatrix} s_{11} & 0 & 0 & 0 \\ 0 & s_{22} & 0 & 0 \\ 0 & 0 & s_{33} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

write down the matrix $(\mathbf{TS})^{-1}$, i.e., write down its sixteen elements. (**10 points**)

5. Given a point $\mathbf{p}$ with *camera space* coordinates $\mathbf{p} = (3, 1, 2)$. In addition, camera space has its origin at $(4, 2, 2)$ in world space, and the basis vectors of camera space have world coordinates $(1, 0, 0)$, $(0, 1/\sqrt{2}, -1/\sqrt{2})$, and $(0, 1/\sqrt{2}, 1/\sqrt{2})$. What are the *world space* coordinates of $\mathbf{p}$?
8 points

6. Derive the 3×3 homogeneous matrix that achieves the transformation shown in the figure.
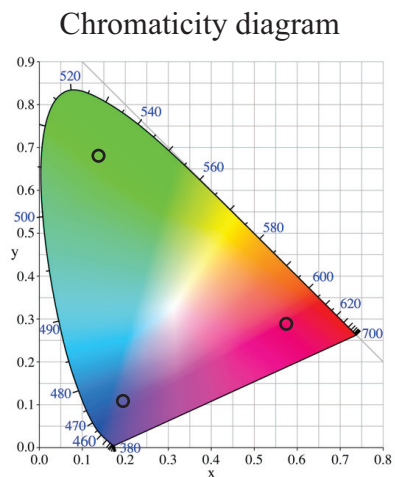   **10 points**
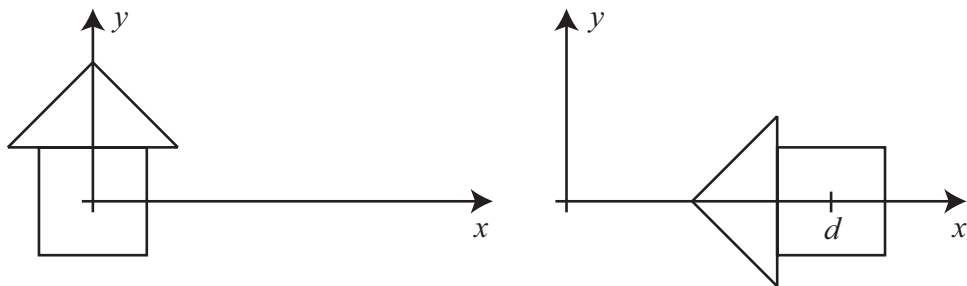
7. Derive a 2×2 homogeneous transformation matrix $\mathbf{M}$ that maps the interval (a, b), where $a, b \in \mathbf{R}$ to the interval (c, d), where $c, d \in \mathbf{R}$. More precisely, find $\mathbf{M}$ such that

$$\mathbf{M} \begin{bmatrix} a \\ 1 \end{bmatrix} = \begin{bmatrix} c \\ 1 \end{bmatrix}, \quad \text{and} \quad \mathbf{M} \begin{bmatrix} b \\ 1 \end{bmatrix} = \begin{bmatrix} d \\ 1 \end{bmatrix}. \quad \textbf{8 points}$$

8. Describe the tristimulus experiment performed by the CIE to define the CIE RGB matching curves $r(\lambda)$, $g(\lambda)$, and $b(\lambda)$. Given the matching curves, how do you determine the rgb-values for a stimulus (i.e., an energy spectrum) $L(\lambda)$? (4-6 sentences) **10 points**

9. Given the choice between two RGB color displays with different sets of RGB primary colors as shown in the figure, which one would you prefer, and why? The primary colors are indicated by the black circles. (3-4 sentences) **8 points**

Monitor A
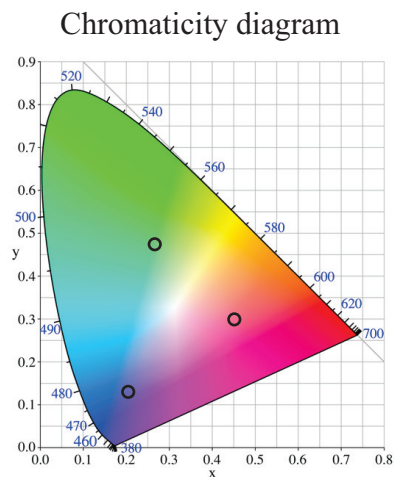
Monitor B

10. Why does linear interpolation of texture coordinates in screen space lead to artifacts? Explain using a skectch and two or three explanatory sentences. **8 points**

11. The Blinn shading model is given by the expression

$$c = \sum_i c_{l_i} \left(k_d \left(\mathbf{L}_i \cdot \mathbf{n} \right) + k_s \left(\mathbf{h}_i \cdot \mathbf{n} \right)^s \right) + k_a c_a.$$

Explain the meaning of all the terms (i.e., c, i, c_{l_i}, k_d, etc.) in this equation. Mention for each term if it is a scalar value, a geometric vector, or if it represents a color. What values would you choose for the relevant parameters to obtain a blue material with a white specular highlight? How do you make the highlight sharper, how do you make it wider? **10 points**

12. Explain why trilinear mipmapping is subject to a trade-off between blurriness and aliasing. What could be done to achieve better antialiasing than trilinear mipmapping? (3-4 sentences)
8 points