CSE167, Introduction to Computer Graphics
Final Exam, Thursday December 13, 2007

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Please include all steps of your derivations in your answers to show your understanding of the problem. Try not to write more than the recommended amount of text. If your answer is a mix of correct and substantially wrong arguments we will consider deducting points for incorrect statements. There are 14 questions for a total score of 100 points.

Your name:

1. Compute the dot product of vectors $\mathbf{a} = (6, 8, 4)$ and $\mathbf{b} = (9, 12, 6)$. What is the angle between $\mathbf{a}$ and $\mathbf{b}$? **4 points**

2. Given two vectors $\mathbf{b}$ and $\mathbf{c}$ and their cross product $\mathbf{a} = \mathbf{b} \times \mathbf{c}$. What is the cross product $\mathbf{c} \times \mathbf{b}$, $\mathbf{b} \times \mathbf{a}$, and $\mathbf{a} \times \mathbf{a}$? **6 points**

3. Given four homogeneous points $\mathbf{p}_0 = (9, 12, 6, 1.5)$, $\mathbf{p}_1 = (12, 16, 8, 2)$, $\mathbf{p}_2 = (9, 12, 6, 1)$, and $\mathbf{p}_3 = (18, 24, 12, 3)$. All of them except one represent the same 3D point. What is that 3D point, and which of the homogeneous points $\mathbf{p}_0, \ldots \mathbf{p}_3$ represents a different 3D point? **6 points**

4. Derive the inverse of each of the following three matrices:

$$\mathbf{M}_0 = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \mathbf{M}_1 = \begin{bmatrix} 1 & 0 & 0 & d \\ 0 & 1 & 0 & e \\ 0 & 0 & 1 & f \\ 0 & 0 & 0 & 1 \end{bmatrix}, \mathbf{M}_2 = \begin{bmatrix} a & 0 & 0 & d \\ 0 & b & 0 & e \\ 0 & 0 & c & f \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad \textbf{8 points}$$

5. A camera is specified by a center of projection $\mathbf{e} = (1, 0, 0)$, a look at point $\mathbf{d} = (1, 1/\sqrt{2}, 1/\sqrt{2})$, and an up vector $\mathbf{up} = (0, 1, 0)$. All these vectors are specified in world coordinates. Derive the camera-to-world and the world-to-camera transformation matrices. **8 points**

6. To rasterize triangles using a multi-level hierarchy one needs to test quickly whether quadrilateral tiles overlap with the triangle. Give pseudo-code for a function *test_quad(v0,v1,v2,v3)* that takes as an input the four vertices of a quadrilateral tile. The function should quickly and conservatively determine whether the tile is guaranteed *not* to overlap with the triangle. *Conservatively* means that the algorithm never misses a tile that overlaps with the triangle. There may be tiles, however, that do not overlap with the triangle but are not detected as such.

   For your convenience, assume that you have a utility function *test_point(edge, point)* that takes as an input an *edge* of the projected triangle and a *point* in the image plane. The function returns 1 if the point lies on the same side of *edge* as the third triangle vertex (i.e., the vertex opposite of *edge*), and 0 otherwise. **8 points**

7. Given a perspective view frustum bounded by four planes. All planes go through the origin. Their outward pointing normals are

$$\begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix}, \begin{bmatrix} -1/\sqrt{2} \\ -1/\sqrt{2} \\ 0 \end{bmatrix}, \begin{bmatrix} -1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \end{bmatrix}, \begin{bmatrix} -1/\sqrt{2} \\ 0 \\ -1/\sqrt{2} \end{bmatrix}.$$

Does the sphere centered at $(1, 2, 3)$ with radius 1 intersect this view frustum? **8 points**

8. Could there be animals with eyes that can distinguish more colors than human eyes? If yes, how would these eyes be different from human eyes? **8 points**

9. What is the main property of a perceptually uniform color space? **4 points**

10. The Blinn shading model is given by the expression

$$c = \sum_i c_{l_i} \left(k_d \left(\mathbf{L}_i \cdot \mathbf{n} \right) + k_s \left(\mathbf{h}_i \cdot \mathbf{n} \right)^s \right) + k_a c_a.$$

Explain the meaning of all the terms (i.e., c, i, c_{l_i}, k_d, etc.) in this equation. Mention for each term if it is a scalar value, a geometric vector, or if it represents a color. Which term is usually the color of the surface? **8 points**

11. Sketch the linear, quadratic, and cubic Bernstein polynomials. List two important properties of these polynomials and explain why they are important for curve design. **8 points**

12. Given a cubic Bézier curve $\mathbf{x}(t)$, which is defined by control points $\mathbf{p}_0, \mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3$. Derive the equation $\mathbf{x}'(t)$ for the tangent of the curve at each value of t. Evaluate $\mathbf{x}'(0)$ and $\mathbf{x}'(1)$.

    Remember that the cubic Bernstein polynomials are

    $$b_0(t) = (1-t)^3, b_1(t) = 3t(1-t)^2, b_2(2) = 3t^2(1-t), b_3(t) = t^3. \quad \textbf{8 points}$$

13. A bilinear surface patch is specified by four control points $\mathbf{p}_{(0,0)} = (0, 0, 0)$, $\mathbf{p}_{(1,0)} = (6, 0, 6)$, $\mathbf{p}_{(0,1)} = (2, 4, 4)$, and $\mathbf{p}_{(1,1)} = (5, 4, 4)$. Write down an equation of the form $\mathbf{x}(u, v)$ that computes a point $\mathbf{x}$ on the surface patch given parameter values $0 < u, v < 1$. Compute two tangent vectors and the unit surface normal at $(u, v) = (2/3, 1/2)$. Evaluate these vectors numerically.
8 points

14. Describe the shadow volume algorithm using a 2D sketch and a few explanatory sentences. You do *not* need to explain how the algorithm is implemented using the stencil buffer. List two disadvantages of shadow volumes compared to shadow mapping. **8 points**