CSE167, Introduction to Computer Graphics
Final exam, Thursday June 12

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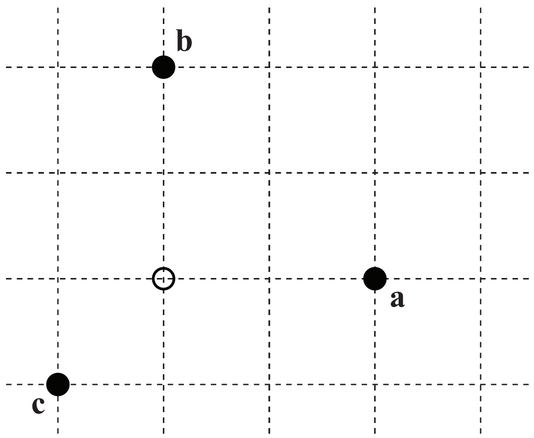
Please include all steps of your derivations in your answers to show your understanding of the problem. Try not to write more than the recommended amount of text. If your answer is a mix of correct and substantially wrong arguments we will consider deducting points for incorrect statements. There are sixteen questions for a total score of 100 points.

Your name:

1. Given four homogeneous points $\mathbf{p}_0 = (3, 4, 2, 0.5)$, $\mathbf{p}_1 = (24, 32, 16, 4)$, $\mathbf{p}_2 = (9, 12, 6, 1)$, and $\mathbf{p}_3 = (18, 24, 12, 3)$. All of them except one represent the same 3D point. What is that 3D point, and which of the homogeneous points $\mathbf{p}_0, \ldots \mathbf{p}_3$ represents a different 3D point? **2 points**

2. Given three vertices in the 2D plane $\mathbf{a} = (5, 4)$, $\mathbf{b} = (3, 6)$, $\mathbf{c} = (2, 3)$. A 2D point $\mathbf{p}$ can be expressed in barycentric coordinates $\mathbf{p} = \alpha\mathbf{a} + \beta\mathbf{b} + \gamma\mathbf{c}$, where $\alpha + \beta + \gamma = 1$. In the figure below, sketch the regions where $\alpha = 0$, $\alpha = 1$, $\beta = 0$, $\beta = 1$, $\gamma = 0$, $\gamma = 1$, and $0 < \alpha, \beta, \gamma < 1$. Compute the α-coordinate for point $\mathbf{p} = (3, 4)$. **6 points**

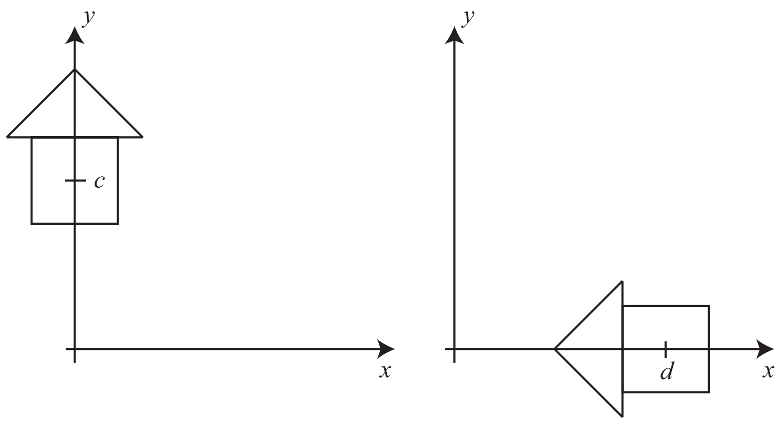


3. Assume you are working with three coordinate systems: object, world, and camera space. The basis vectors of object space have world coordinates $(1, 0, 0), (0, 0, -1)$, and $(0, 1, 0)$. The origin of object space has world coordinates $(0, 0, 10)$. The basis vectors of camera space have world coordinates $(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3}), (-1/\sqrt{2}, 0, 1/\sqrt{2})$, and $(1/\sqrt{6}, -2/\sqrt{6}, 1/\sqrt{6})$. The origin of camera space has world coordinates $(5, 9, -4)$.

   Write down the 4×4 matrices that transform object to world coordinates, camera to world coordinates, world to camera coordinates, and object to camera coordinates. For the object to camera coordinate transformation, you do not need to multiply out the matrix product.

   What are the camera coordinates of the origin of object space? **8 points**

4. Derive the 3×3 homogeneous matrix that achieves the transformation shown in the figure. In addition, compute its inverse. **8 points**



5. What is the purpose of the 4×4 homogeneous projection matrix? In particular, how is the projection matrix related to the view frustum? **6 points**

6. Explain what a z-buffer is, how z-buffering works, and what its purpose is. **6 points**

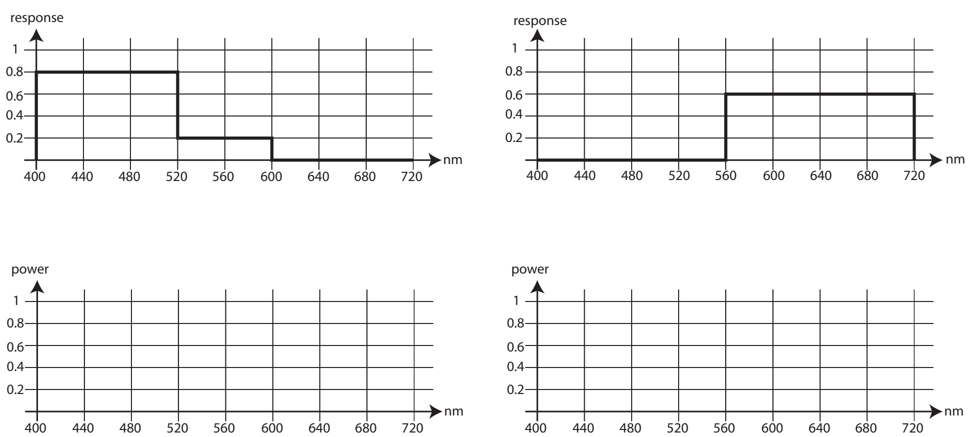
7. Give an intuitive explanation of how aliasing artifacts occur in texture mapping. You may use an example or draw a sketch. The sketch may show a one-dimensional function to illustrate the general problem. Describe the main idea of antialiasing techniques such as trilinear mip-mapping. **6 points**

8. Explain how per-object view frustum culling works and what its main benefit is. Give an example of a scenario where it is particularly beneficial. **6 points**

9. A plane is given by its normal $\mathbf{n} = (-1/\sqrt{2}, 0, -1/\sqrt{2})$ and a point on the plane $\mathbf{p} = (4, 8, 3)$. Write down a function $d(\mathbf{x})$ that computes the distance of a point $\mathbf{x}$ to the plane. Does a sphere centered at $\mathbf{x} = (1, 1, 4)$ with radius $r = 1.5$ intersect the plane? **6 points**

10. Explain the term *metamer*. **3 points**

Assume that an animal has two types of color receptors with response curves shown in the figure below. Sketch two spectral distributions that are metamers for this animal. Use the two empty graphs below. Hint: There are many different possibilities. Start by thinking about how the response to a spectrum is computed. **3 points**



11. Shading of diffuse surfaces is based on Lambert's cosine law. Describe the distribution of light reflected by diffuse surfaces. Explain Lambert's law and how it is derived. You may find it useful to draw a sketch. **6 points**

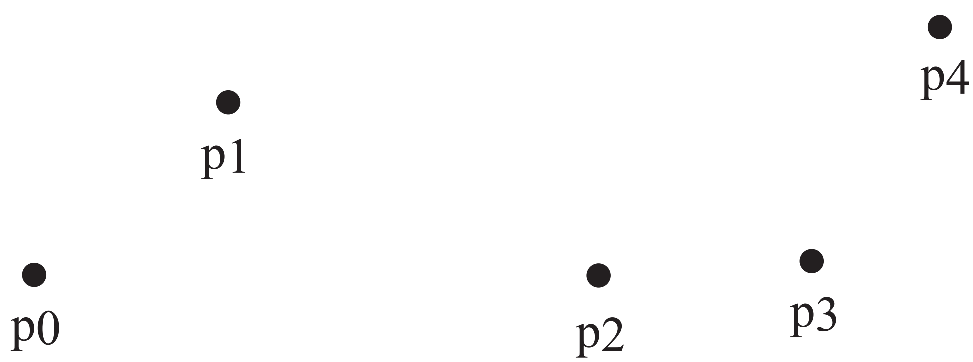
12. A point light at position $\mathbf{p}$ is illuminating a point $\mathbf{x}$ on a surface. Explain why the strength of the light is proportional to $1/\|\mathbf{p} - \mathbf{x}\|^2$, i.e., one over the squared distance between the light and the point on a surface. Does this rule also apply to spot lights? What about directional lights? **6 points**

13. Write down pseudo-code for a vertex and fragment shader that perform per pixel diffuse shading for a single light source. The input to the vertex shader is the vertex position, normal, and diffuse color, and the light position and color. You can use your own names for these variables, there is no need to use the OpenGL names. List one advantage and one disadvantage of computing shading on a per pixel basis compared to shading on a per vertex basis.**8 points**

14. Describe the shadow mapping algorithm using a sketch and a few explanatory sentences. List two potential problems or artifacts that may appear with shadow mapping. **6 points**

15. The figure below shows the control points of a single segment of a Bézier curve. What is the degree of the curve? Sketch the evaluation of the curve at (approximately) $t = 0.75$ using the de Casteljau algorithm.

    Sketch the convex hull of the control points. Explain the convex hull property of Bézier curves. **6 points**



16. A bilinear patch $\mathbf{x}(u, v)$ is given by four control points $\mathbf{p}_0 = (2, 0, 1)$, $\mathbf{p}_1 = (4, 2, 1)$, $\mathbf{p}_2 = (2, 2, 0)$, and $\mathbf{p}_3 = (7, 4, 5)$; and $\mathbf{x}(0, 0) = \mathbf{p}_0$, $\mathbf{x}(1, 0) = \mathbf{p}_1$, $\mathbf{x}(0, 1) = \mathbf{p}_2$, and $\mathbf{x}(1, 1) = \mathbf{p}_3$. Evaluate the patch at $(u, v) = (2/10, 5/10)$. In addition, compute the tangent vectors and the normal at this point. **8 points**